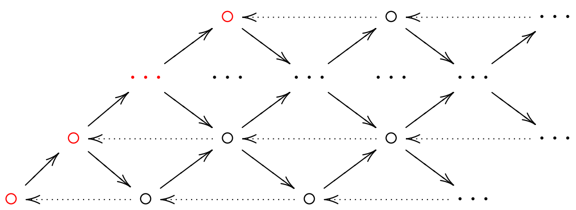
Chapter 1

mth power algorithm

We introduce here an algorithm which calculates the connected components of $(\Gamma^1_{A_{nm-1}})^m$. This algorithm is implemented in the programming language Python. It takes as argument the natural numbers n and m and outputs the list of connected components of $(\Gamma_{A^1_{nm-1}})^m$, each of the components being given as a an ordered list of sectional paths. The construction of the algorithm is based on the following remarks:

1. Each connected component is isomorphic to $\mathbb{Z}A_{n'}/\varphi$ where $n' = n$ or $n-1$ and φ is the automorphism corresponding to the automorphism of $\tilde{\Gamma}_{A_{nm-1}}$, $\eta_{\gamma m}$ for some $\gamma \in \mathbb{Z}$, that is, it has the general form:



   where the longest sectional paths in $\mathbb{Z}A_{n'}/\varphi$ have the same form as the red sectional path in the above figure.

2. Each component has at least one vertex in the fundamental region Δ_m.

3. Starting at a vertex $(i,j) \in \Delta_m$, there exits an arrow in the mth power of $\Gamma^1_{A_{nm-1}}$, $D : (i,j) \to (i',j')$ if and only if $(i',j') = (i, j+m)$ (see proof of Proposition **??**).

4. Starting at a vertex $(i,j) \in \Delta_m$, the ordered list of vertices

$$(i, j + \alpha(i)m)$$

   (where $0 \leq \alpha(i) \leq \lfloor \frac{nm+1-j}{m} \rfloor$ if $i = 1$ and $0 \leq \alpha(i) \leq \lfloor \frac{nm+2-j}{m} \rfloor$ otherwise) form a longest sectional path in the connected component containing (i, j) (since the component is either isomorphic to $\mathbb{Z}A_n$ or to $\mathbb{Z}A_{n-1}$).

5. In each connected component there are arrows between two sectional paths of 4., X, Y if and only if $\tau^{\pm\alpha m}(Y) = X$ in $\Gamma^1_{A_{nm-1}}$ where $\alpha \in \mathbb{N}$. Moreover given two sectional paths X and Y such that $\tau^{-m}(X) = Y$ and two vertices (i, j), (i', j') such that $(i, j) \in X$ and $(i', j') \in Y$, there is an arrow $D' : (i, j) \to (i', j')$ if and only if $(i', j') = (i + m, j)$.

Hence in order to find the structure of a connected component it suffices to have an ordered list of its longest sectional paths $v_1, v_2, \cdots$ such that

$$\tau^m(v_{i+1}) = v_i.$$

Algorithm 1: Calculation of the connected components of $(\Gamma^1_{A_{nm-1}})^m$

Input: The natural numbers n and m

Output: The set of the connected components of $(\Gamma^1_{A_{nm-1}})^m$, where each connected component is given as an ordered list of its longest sectional paths $v_1, v_2, \cdots$ such that $\tau^m(v_{i+1}) = v_i$.

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Δ_m = set of vertices in the fundamental region of Γ^1_{A_{nm-1}};
𝒞 = ∅, the set which will contain the connected components;
foreach vertex (i_0, j_0) in Δ_m do
    v_ref = ordered list of the vertices of the longest sectional path
    starting at (i_0, j_0);
    C = v_ref, the ordered list which will contain all the longest
    sectional paths of the connected component to which (i_0, j_0)
    belongs;
    (i, j) = (i_0, j_0);
    while All the vertices of v_ref have not appeared do
        v = ordered list of the vertices of the longest sectional path
        starting at (i + m, j + m);
        Add v to C ;
        if one of the vertices (i', j') of v belongs to Δ_m then
            Δ_m = Δ_m \ (i', j');
        end
    end
    𝒞 = 𝒞 ∪ C;
end
return 𝒞;
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