Mind the Gap – Volatile Measurement Bias in the Index of Consumer Prices and Monetary Policy

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Abstract
I argue that the gap between consumer price index (CPI) inflation and actual inflation varies systematically across the business cycle. CPI inflation is the change in expenditure required to afford a fixed basket of varieties. Actual inflation is the change in minimum expenditure required to afford a given level of utility. In a New Keynesian model with variety entry and exit and sticky prices, the source of CPI measurement bias is statistical authority’s failure to account for new varieties in time. The model predicts that the gap between CPI inflation and actual inflation is large in absolute value whenever the economy is far from trend but small otherwise. In particular, CPI inflation overstates actual inflation in times of high productivity. Overall, CPI inflation is more persistent and less volatile when compared to actual inflation. Applying the model to U.S. CPI inflation reveals a volatile measurement bias.

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## Contents

1 Motivation .................................................. 1

2 Model setup ................................................. 3
   2.1 Household ............................................. 3
   2.2 Variety entry and exit ................................. 3
   2.3 Actual price level under variety entry and exit .... 4

3 Measurement biases in CPI inflation ..................... 5
   3.1 CPI inflation .......................................... 6
   3.2 CPI bias and its components ......................... 7
   3.3 Delay bias ............................................ 8
   3.4 Output and output gap ............................... 9

4 Model solution .............................................. 10
   4.1 Equilibrium and steady state ......................... 10
   4.2 Calibration ........................................... 10

5 Results on CPI bias and monetary policy ................ 12
   5.1 Volatile bias in U.S. CPI inflation .................. 12
   5.2 Dynamic adjustment to shocks ...................... 12

6 Conclusion .................................................. 14
1 Motivation

Economic analysis asserts that for monetary policy to maximize welfare, it should stabilize actual inflation which policy makers do not observe.\(^1\) Policy makers observe consumer price inflation which is the change in the consumer price index (CPI). The CPI reflects expenditure required to afford a basket of varieties at current prices relative to expenditure required to afford the same basket at prices in some previous period. Actual inflation is the growth rate of the price level which arises in a utility-maximizing framework. This price level reflects minimum expenditure required to afford a given level of utility.

For welfare-maximizing monetary policy knowledge about the gap between CPI inflation and actual inflation matters. Conceptually, there is easy account of a constant gap between CPI and actual inflation by adjusting policy targets appropriately and many central banks indeed associate price stability with constant small positive instead of zero CPI inflation. However, a volatile gap between CPI and actual inflation complicates the assessment of the current stance of monetary policy (Shapiro and Wilcox (1996)). Unfortunately, there is no easy solution to address volatile measurement bias, a point recently stressed by Ottmar Issing:

The present scenario of rapid changing technology combined with low inflation makes the issue of measurement biases in price indices of the utmost relevance for monetary policy. […] A near constant or low volatile bias is not of major concern […] A high volatility of the bias is a matter of much more serious concern, and makes achieving price stability more difficult. There is no clear indication on the size of this volatility. There is a need therefore to enhance our knowledge on how best to conduct monetary policy in the presence of a highly volatile measurement bias. Issing (2001, p.2 and 4)\(^2\)

In this paper, I argue that the gap between CPI and actual inflation varies systematically with the business cycle. Accordingly, data quality of the CPI is good in certain states of the business cycle and bad in others. The argument is developed in a dynamic general equilibrium New Keynesian model with variety entry and exit and sticky prices where a statistical authority compiles the CPI based on Laspeyres’ index. In the model two measurement biases exist. One bias is the well-known substitution bias. It arises because the optimizing households reacts to relative price changes by changing quantities. The CPI cannot capture such substitution since quantities attached to prices are fixed. The other bias is due to the failure of the statistical authority to include new varieties in its market basket in time. Diewert (1999) argues for the prominence of this bias in the U.S. CPI.

The main results are as follows. The gap between CPI inflation and actual inflation is large in absolute value whenever the stance of the business cycle is far from trend.

\(^1\)See Woodford (2002) for inflation stabilization and welfare.
\(^2\)The relevance of reliable statistical systems for monetary policy making in the euro area. Speech by Professor Otmar Issing, Member of the Executive Board of the European Central Bank, CEPR/ECB. Workshop on issues in the measurement of price indices, Frankfurt am Main, 16th November 2001.
More precisely, if productivity is high (low) CPI inflation overstates (understates) actual inflation and thus reinforces concerns articulated by policymakers (Issing (2001)). Time-variation in measurement bias is exclusively due to the delay bias. That is, in the linearized model the substitution bias is negligible. The fundamental reason for time-variation in the delay bias is that the statistical authority tracks a non-representative array of varieties due to delayed recording of new varieties. Non-representativeness matters most when prices of new varieties take values very different from the average price. This happens exactly if the business cycle takes extreme states.

Technically, measured inflation depends on contemporaneous actual inflation and on lags of actual inflation. Therefore one perspective on measurement bias is to consider it as a filter that once applied to actual inflation recovers CPI inflation. Conversely, the inverse filter recovers actual inflation once applied to CPI inflation. The frequency domain reveals that the filter shifts weight of the spectrum of measured inflation to high frequencies and increases variance of measured inflation across all frequencies. Thus one finds that CPI inflation is more persistent and less volatile than actual inflation. The model lends itself to recover historic paths of actual inflation. I demonstrate this application for U.S. CPI inflation. Measurement bias in U.S. CPI inflation is volatile, sizeable and correlates negatively with U.S. CPI inflation. By the construction of data for real economic activity measurement bias in CPI inflation translates into opposite bias in real output. Accordingly, even under the extreme assumption that a theory-consistent measure of the output gap is available it remains a distorted measure of economic activity. Moreover, measurement bias in inflation and real output (gap) are interdependent and not independent stochastic processes as is the basic presumption in recent contributions to the literature on robust monetary policy rules.

Under the assumption that monetary policy refers to measured variables delayed recording of new varieties translates into dampened responses of measured variables to aggregate cost push shocks when compared to actual quantities. Differences between measured and actual variables are sizable for inflation and large for the output gap. Measurement bias is persistent for inflation and very persistent for the output gap. Shapiro and Wilcox (1996) note that very few paper have attempted to investigate empirically time series properties and volatility of CPI bias, mainly due to data limitations. Therefore, the model here may proof a useful handle on volatile and hard to observe bias in CPI inflation.3

The remainder of the paper is organized as follows. Section 2 sets out the basic model. Section 3 describes compilation of CPI inflation and the two sources of measurement bias. Section 4 provides details of the model solution. Section 5 recovers measurement bias in U.S. CPI inflation and analyzes measurement bias in response to shocks in general equilibrium.

3Greenwood and Uysal 2004 note that economic models provide a useful laboratory for assessing the performance of alternative price index measures.
2 Model setup

In this section, I derive a stylized New Keynesian model with variety entry and exit. The model is a minimal setup and conveys the basic intuition for time-variation in CPI measurement bias. As a special case, the model reduces to the standard New Keynesian model without entry and exit in Woodford (2003).

2.1 Household

A representative household is endowed with time, its ownership of firms and last period bond holdings. It chooses infinite sequences of varieties \( c_t(n) \), labor \( L_t \) and nominal bond holdings \( B_t \) to maximize expected discounted lifetime utility,

\[
\max \{c_t(n), B_t, L_t\} \quad \text{subject to} \quad \int_0^1 p_t(n) c_t(n) \, dn + B_t = i_{t-1} B_{t-1} + W_t L_t + D_t + T_t.
\]

The function \( u(.) \) \( (h(.)) \) is twice continuously differentiable, increasing in its argument and concave (convex) and \( \xi_t \) is a shock to dis-utility of labor. Variety \( n \) is consumed at amount \( c_t(n) \) and sells at price \( p_t(n) \). The household receives nominal income out of bond investment, labor income, nominal dividends \( D_t \) and lump-sum subsidies \( T_t \). \( W_t \) and \( i_t \) represent the nominal wage rate and the nominal interest rate, respectively. Bonds are traded among households. Table 2 describes parameters. The household bundles varieties according to the Dixit-Stiglitz technology

\[
C_t = \left( \int_0^1 c_t(n)^{\frac{\theta-1}{\theta}} \, dn \right)^{\frac{1}{\theta}},
\]

with \( \theta > 1 \). The utility-based or actual price level measures the minimum expenditure required to obtain a marginal unit of the consumption aggregate \( C_t \),

\[
P_t = \left( \int_0^1 p_t(n)^{1-\theta} \, dn \right)^{\frac{1}{\theta-1}}.
\]

2.2 Variety entry and exit

Variety \( n \in [0, 1] \) is produced by firm \( n \). Thus, there is a one to one mapping between a variety and its firm. The unit interval comprises \( \delta \in [0, 1) \) new firms and \( (1-\delta) \) established firms. New firms produces a variety for the first time, established firms produces their variety at least for the second time. All firms produce with identical technology

\[
y_t(n) = Z_t L_t^\phi(n), \quad \phi \in (0, 1),
\]

where firm-specific hours \( L(n) \) are the sole input obtained from a competitive factor market and \( Z \) denotes aggregate productivity. In every period an exogenously given mass

\[4\text{In a companion paper I set out a version of the model with endogenous entry.}\]
\( \delta/(1-\delta) \) of newcomers out of a large pool is successful in entering the economy. Newcomers face a time-to-build period before they start production. At the end of each period, an exit shock causes a fraction \( \delta \) of firms and newcomers to leave the market. All shocks are small such that no firm ever liquidates owing to bad shocks. The representative household owns all firms and is entitled to firm dividends.

### 2.3 Actual price level under variety entry and exit

Provided all varieties are priced identically and deliver identical utility to the household, price level measurement is trivial: it suffices to observe a single price to reveal the actual price level. Only a non-degenerated distribution of prices makes the problem of price level measurement interesting. To this end, I assume that established firms set prices according to Calvo (1983). The probability of an established firm to adjust its price in a given period equals \((1-\alpha)\) where \(\alpha \in [0,1)\). A new firm chooses its price optimally with probability one. It therefore faces a problem identical to the problem of an established firm which happens to re-optimize its price in the current period. In subsequent periods the former new firm is subject to the standard Calvo mechanism.\(^5\) The optimal price of firm \(n\) solves

\[
\max_{p_t(n)} \mathbb{E}_t \sum_{s=t}^{\infty} (\kappa \beta)^{s-t} \sum_{s,t} \left\{ \frac{p_t(n) y_s(n)}{P_s(1-\tau_s)^{-1} - W_s \frac{y_s(n)}{Z_s}} \right\} \quad \text{s.t.} \quad \frac{y_s(n)}{C_s} = \left( \frac{p_t(n)}{P_s} \right)^{-\theta}
\]

with \(\Sigma_{s,t} = \frac{u_C(C_t)}{u_C(C_t)}\) the stochastic part of the discount factor of the representative household for future real cash flows, \(\kappa = \alpha(1-\delta)\) and \(\tau_s\) denoting proportional sales taxes. The optimality condition reveals that all optimizing firms face identical problems and hence set identical optimal price \(p_t\).

A crucial assumption is that firms set their price with reference to the actual price level. In the real world firms are likely to orientate their price mainly at an index of prices in their particular industry. Observability of the actual price level reflects the notion that firms in general will be extremely well informed about prices of competitors when compared to the degree of information of a statistical authority that has the task to compile a price index for the entire universe of varieties.\(^6\) Combining the optimal price with the actual price level establishes the following proposition.

**Proposition 1:** The utility-based price level \((4)\) has recursive representation

\[
P_t^{1-\theta} = (1-\kappa) p_t^{1-\theta} + \kappa P_{t-1}^{1-\theta},
\]

where \(p_t\) denotes the price optimally chosen at date \(t\) and \(\kappa = \alpha(1-\delta)\).

\(^5\)By exogenous exit the possibility is excluded that a firm increases profits by exiting the market in one period for the sole reason to enter the market in a following period then exploiting the possibility to set a new price.

\(^6\)Boivin, Giannoni and Mihov (2007) argue that sectoral price indices react quickly to sector-specific shocks but sluggishly to aggregate shocks. Mackowiak and Wiederholt (2006) analyze what information firms actively choose to observe and find that firms track idiosyncratic shocks much more intensive than aggregate shocks.
In any period, variety entry and exit increases the fraction of varieties with optimal price and, at the same time, decreases the relevance of the weighted average of past optimal prices. To obtain this result it is instructive to consider the price distribution of an arbitrary entry cohort that entered the economy in period \( s \). Under time-dependent pricing, such a price distribution is truncated Poisson and summarized by

\[
\Lambda_t(s) = (1 - \alpha)^{s+1} \sum_{k=t}^{s+1} \alpha^{t-k} p_k^{1-\theta} + \alpha^{t-s} p_s^{1-\theta}, \quad s < t.
\]

If \( s = t \) this reduces to \( \Lambda_t(t) = p_t^{1-\theta} \). Summing over all cohort distributions weighted by cohort size summarizes the full price distribution. The remaining part is to show that \( P_t \) has recursive representation.

It is then straightforward to derive an approximate New-Keynesian Phillips curve by combining the linearized optimal price with the linearized recursive law of motion of the actual price level,

\[
\pi_t = \beta E_t \pi_{t+1} + \sigma x_t + u_t.
\]

The error term \( u_t \) is proportional to \( \tau_t \equiv d \tau_t / (1 - \tau) \). The approximation is taken around a zero-inflation steady state with \( \pi_t \equiv d \Pi_t / \Pi \) and \( \Pi_t = P_t / P_{t-1} \). The output gap \( x_t \) is defined as sticky-price output minus output in efficient equilibrium. Moreover,

\[
\sigma = \frac{[1-\alpha(1-\delta)]\beta}{\alpha(1-\delta)} \zeta, \quad \zeta = \frac{\eta + (\gamma - 1)\phi + 1}{\phi\delta(1-\phi)}.
\]

It turns out that \( \sigma \) is increasing in \( \delta \) but decreasing in \( \alpha \). The more flux of firms, the more important becomes the output gap for inflation dynamics because in every period a larger fraction of firms set prices optimally as a function of current marginal cost. Higher \( \alpha \) in turn increases the extent to which firms are forward looking and thus reduces the effect of current marginal cost.\(^7\)

An important implications of variety entry and exit is that the average censored price duration \( (1 - \alpha(1 - \delta))^{-1} \) decreases in the exit probability since variety exit increases the likelihood for price contracts to terminate in the future. This is in line with the finding in Bils and Klenow (2004) based on a large array of consumer goods that the rate of product turnover is a robust predictor of more frequent price changes.

### 3 Measurement biases in CPI inflation

The market introduction of new varieties decreases coverage of a market basket, such as the one underlying the CPI, if this basket records new varieties with delay. Low coverage

\[^7\]To highlight the implications of variety entry and exit for the duration of price contracts consistent with a particular estimate of \( \sigma \) consider

\[d \sigma = \frac{\partial \sigma}{\partial \delta} d \delta + \frac{\partial \sigma}{\partial \alpha} d \alpha = 0 \quad \text{or} \quad \frac{d \alpha}{\partial \alpha} = -\frac{\partial \sigma}{\partial \delta} / \frac{\partial \sigma}{\partial \alpha} \geq 0.
\]

Thus, higher variety entry and exit requires a larger price duration \( (1 - \alpha)^{-1} \) to maintain a particular \( \sigma \). The reason is that entry and exit provides additional reason for price contracts to terminate. To compensate for this effect \( \alpha \) has to increase.
is an important reason for measurement bias to occur. Historically, several years pass between two consecutive base periods of the CPI.\footnote{For the U.S., the historical record of revisions is 1940, 1953, 1964, 1978, 1987 and 1998. Until 1998, weights were usually outdated by roughly three years when they entered the base period revision. Starting with 2002 weights are now updated bi-annually (table 1 in BLS Handbook of Methods, chapter 17).} In base periods, weights are determined according to which prices of available varieties enter the index. Accordingly, varieties that enter the market between base periods obtain zero weight and the coverage of the index decreases at the rate of variety entry after each base period.

Statistical authorities have recognized the problem of decreasing coverage for long and have developed methods to account more promptly for varieties that arrive after the last index revision. The main method implemented by the U.S. Bureau of Labor Statistics for the U.S. CPI is sample rotation. In every year, the market basket is updated for about 20\% of the geographic area such as to better reflect recent developments in consumer expenditure in this particular area. The entire market basket then is updated once every five years (Armknecht, Lane, Stewart 1997).

3.1 CPI inflation

I capture the efforts of statistical authorities to maintain suitable coverage between base periods by imposing a constant time lag $\ell \geq 1$ with which new varieties enter the basket maintained by the statistical authority. Put differently, in the model only those varieties enter the market basket which are present in the market since $\ell$ periods or longer. The information lag mirrors average time elapsed between market and basket introduction of a new variety as induced by sample rotation, delays in obtaining survey data on consumer expenditure, limits in the technology to collect data, current practice to average consumer expenditure data over several years to mitigate influences of seasonality and idiosyncratic shocks or, more generally, limited availability of funds and resources. For example, large scale introduction of modern information technology such as price-scanning would reduce $\ell$.\footnote{There is an independent reason to prefer a constant time lag over a fixed base period. Suppose that coverage decreases between base periods despite efforts of statistical authorities. Unless in empirical exercises decreasing coverage and thereby time-variant data quality is explicitly accounted for using e.g. time-varying coefficients, purely linear models deliver empirical impulse response functions (IRFs) that one may think of as average response where the average is taken over many different regimes of CPI coverage. Thus, IRFs of the model with a constant time lag match conceptually better with empirical IRFs from linear models which are widely available.} Let statistical authority use Laspeyre’s index to compile CPI inflation, $\Pi^m_t \equiv P^m_{t|t} / P^m_{t-1|t}$, where measured price levels read

$P^m_{t|t} = \int_{\mathcal{N}(\ell)} q(n) p_t(n) \, dn , \quad P^m_{t-1|t} = \int_{\mathcal{N}(\ell)} q(n) p_{t-1}(n) \, dn .$

Here $\mathcal{N}(\ell) \equiv [0, (1 - \delta)^\ell]$ is the set of varieties contained in the basket of the statistical authority. If $\ell$ is large and therefore new varieties enter the basket with considerable delay, this also induces low coverage of the entire universe of varieties.Measured quantities are $q(n) = (1 - \delta)^{-\ell}$.\footnote{Here, I restrict attention to time-invariant quantities which allows me to discuss properties of measured price level relative to actual price level below. Generalization to a setup with time-variant weights is straightforward but does not alter basic conclusions in a linearized model.} It is instructive to note that the basket $\mathcal{N}(\ell)$ is updated in every period.
The update accounts for varieties that first were marketed \( \ell \) periods ago and excludes varieties that left the market at the end of the previous period. The second \( t \)-subscript of \( P^m \) reflects the updating period.\footnote{Because firm entry and exit is exogenous \( N(\ell) \) is not a function of time. This assumes that new varieties obtain indices \( n \in [0, 1] \) of varieties that exited in the previous period.} Period-by-period updating of the market basket again is in line with the sample rotation scheme implemented by the U.S. BLS.

### 3.2 CPI bias and its components

A natural benchmark against which to judge the accuracy of CPI inflation is the gross growth rate of the utility-based price level \( \Pi_t \). This benchmark ensures comparability to the vast empirical literature on average CPI-bias (see among many others, Feenstra and Shiells (1997)). Moreover, some statistical authorities view their index as approximation to the true cost-of-living index.\footnote{For example, this is the case for the U.S. BLS (Armknecht et al. 1997, p.388), but is not the case for the Office for National Statistic in the United Kingdom, see chapter 10.10, CPI Technical Manual 2005.}

Most importantly, however, the bulk of normative conclusions about optimal monetary policy derived in micro-founded models implies (or is even formulated as) a particular path of \( \Pi_t \). If one agrees with the presumption that, in case of bias, \( \Pi^*_t \) matches observed CPI inflation better than \( \Pi_t \) it should be of considerable interest to learn how paths of \( \Pi_t \) translate into paths of \( \Pi^*_t \).

I define CPI measurement bias as CPI inflation over actual inflation, 
\[
B_t = \left( \frac{\int N(\ell) w^m_t(n) \pi_t(n) \, dn}{\left( \int N(\ell) w_t(n) \pi_t(n) \right)^{1-\theta}} \right) \times \left( \frac{\int N(\ell) p^1_t(n) \, dn}{\left( \int N(\ell) p^{1-\theta}_t(n) \, dn \right)^{1-\theta}} \right)^{-1}
\]

\[
= [SB_t] \times \{WB_t\}.
\]

The variable \( \pi(n) \) captures inflation of variety \( n \) and \( w^m(n) \) \( (w(n)) \) denotes measured (actual) nominal consumption expenditure spend on variety \( n \) relative to total consumption expenditure.

Consider first the substitution bias. It arises because changes in relative prices lead the optimizing household to replace the variety with high price to some extent by the variety with low price. But Laspeyre’s index cannot capture this substitution since it fixes quantities attached to prices.

**Proposition 2:** Up to first order, the substitution bias as defined in equation (8) is zero.

Intuitively, first order approximation decouples all variables, in particular variety-inflation rates and corresponding weights. Because weights integrate to unity by construction, their percentage deviations from steady state integrate to zero. The remaining difference between the nominator and the denominator in \( SB_t \) is curvature which again is
irrelevant up to first order. One immediate implication of proposition 2 is that the delay bias equals total bias.

**Proposition 3:** Up to first order, the delay bias is a finite-order lag polynomial of actual inflation,

\[
\hat{B}_t = \frac{a\delta}{1-a(1-\delta)}\pi_t + \sum_{k=1}^{\ell-1} \frac{(1-a)\kappa^k}{1-a(1-\delta)}\pi_{t-k} = (a(L) - 1)\pi_t. \tag{9}
\]

Here \(L\) denotes the lag-operator. Then CPI inflation is a finite order lag polynomial of actual inflation,

\[
\pi^m_t = a(L)\pi_t. \tag{10}
\]

Two observations are noteworthy. First, the order of \(a(L)\) directly relates to the information lag of the statistical authority. Second, if actual inflation varies in equilibrium, in general the CPI bias also varies. This is true despite the fact that variety entry and exit is exogenous and time-invariant.

There are two informative special cases when CPI bias vanishes and CPI inflation equals measured inflation. First, CPI bias is zero if variety entry is zero, i.e. \(\delta = 0\). This case coincides with the basic New Keynesian model without variety entry and exit. Zero entry and exit closes the extensive margin and thus renders the informational constraint of the statistical authority irrelevant. Second, CPI bias is zero if prices are flexible, i.e. \(\alpha = 0\). In this case all prices are identical and since the distribution of prices collapses the problem of tracking this distribution becomes trivial: observing an arbitrary price in any given period is sufficient to infer the actual price level.

### 3.3 Delay bias

To illustrate the source of the delay bias, I discuss the difference between measured and actual price distribution and show how CPI inflation translates into actual inflation. In the linearized model, measured and actual price levels connect according to

\[
\hat{p}^m_{t|t} = (1-\alpha) \sum_{k=0}^{\ell-1} \alpha^k \hat{p}_{t-k} + \alpha^\ell \hat{P}_{t-\ell}.
\]

Because the statistical authority fails to record prices of varieties newly arrived during the last \(\ell\) periods, measured fractions of optimal prices deviate from actual fractions \((1-\kappa)\kappa^k\) in these periods. Figure 1 plots measured and actual fractions for different \(k\). The statistical authority effectively underestimates the relevance of recent optimal prices. Different weighting schemes matter most whenever recently optimized prices differ a lot from past optimal prices. Thus, CPI inflation underestimates actual inflation (i.e. bias is negative) whenever actual inflation is large because large actual inflation indicates strong movements in recently optimized prices.\(^{13}\) Transforming price levels into inflation rates, CPI inflation becomes the finite-order lag-polynomial of actual inflation as in proposition 3.

\(^{13}\)This relationship is obtained by reformulating the actual price level as \(\hat{p}_t - \hat{P}_t = \frac{\kappa}{1-\pi}\pi_t\).
Figure 1: Measured and actual price distribution. The first joint bar indicates that in the current period the statistical authority estimates that 32.7% of varieties fare at the optimal price, whereas actually 27% of varieties fare at the optimal price.

3.4 Output and output gap

Bils (2004) argues that for the U.S. any bias in CPI inflation converts into a composite bias in real growth rates because most consumption deflators for the National Income and Product Accounts (NIPA) are based on measures of CPI inflation produced by the BLS. The NIPA then derives real measures of growth by subtracting from growth in nominal expenditure for each category of goods the respective measure of inflation.

In line with the NIPA approach I deflate nominal expenditure using the measured price level. Let $Y_t$ denote actual real output where individual varieties are aggregated according to an index identical to (3). In the absence of government demand for individual varieties and without investment actual real output equals consumption and the GDP deflator coincides with the utility-based price level. The estimate of actual real output obtained by the statistical authority is $Y_t^m \equiv P_t Y_t / P_t^m$. Measured real output in the linearized model readily computes

$$\hat{Y}_t^m = \hat{Y}_t + b(L) \pi_t$$

$$b_j = \frac{\alpha j + \delta}{\alpha (1 - \delta)} \quad j = 0, \ldots, \ell - 1 .$$

Here $b_j$ denotes the coefficients of the finite-order lag-polynomial $b(L)$. It is convenient to convert measured output into the measured output gap,

$$x_t^m = x_t + b(L) \pi_t .$$
The measured output gap \( x_m^t \) is the difference between measured output under sticky prices minus the efficient level of output which is free of measurement bias due to absence of price rigidities.

In the literature, several explanations motivates the assumption of persistent misperception of the unobservable output gap on the side of central banks. Orphanides (2003) shows that flash estimates of macroeconomic data often undergo major revisions as statistical authorities account for more complete-source information over time.\(^{14}\) Such data revisions then result in revisions of the output gap economists try to now-cast. Smets (2001) assumes that only the output gap moves inflation and that the central bank has timely and accurate data on past and current real GDP. Because real GDP decomposes into potential output and the output gap the central bank faces a signal extraction problem: on the basis of its information it cannot distinguish between movements in potential output and the output gap.

In contrast, the fundamental source of the bias in \( x_m^t \) considered here is a conceptual one. It originates in the use of Laspeyre’s index formula for the aggregation of individual prices under variety entry and exit and transmits with the compilation of real growth rates into real output and the output gap. Thus even with finally-revised data and even if the central bank perfectly discriminates between measured output and potential output this measurement bias continues to exist.

4 Model solution

4.1 Equilibrium and steady state

Given initial conditions and stochastic sequences \((\varepsilon_t, u_t)_{t=0}^{\infty}\), an equilibrium is an infinite sequence of quantities and prices such that (i) the household maximizes life time utility (1) subject to (2) and (3), (ii) firms choose prices according to (5), (iii) the monetary authority conducts policy (see below), (iv) goods markets and the labor market and the bond market clear.

I consider a steady state in which prices are free to adjust at any time. Utility-based price level and optimal pricing imply that prices of all firms are identical and equal to the actual price level, \( P = p_n \). Calibrating \( L \) and solving for \( \zeta \) instead, one obtains aggregate consumption \( C = ZL^g \). Equating the marginal rate of substitution between labor and consumption to optimal pricing then delivers the unique solution for \( \zeta \). It is straightforward to show that equilibrium profits are positive as long as \( \tau \) is smaller than unity. From this it follows that the value of an entry candidate is positive.

4.2 Calibration

The linearized model is summarized in table 1.\(^{15}\) I follow Sims (2002) to solve for the

\(^{14}\)Historically, however, CPI inflation is a time series that is extremely rarely revised.

\(^{15}\)I abstract from a non-zero mean of \( \varepsilon_t \). It is an interesting extension to consider the effect of volatile CPI bias on policy targets.
\[ \pi_t^m = a(L)\pi_t \]
\[ x_t^m = x_t + b(L)\pi_t \]
\[ \pi_t = \beta E_t\pi_{t+1} + \sigma x_t + u_t \]
\[ x_t = E_tx_{t+1} - \gamma^{-1}(\hat{t}_t - E_t\pi_{t+1}) + \epsilon_t. \]

Table 1: Linearized model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta)</td>
<td>0.0797</td>
<td>Entry and exit rate</td>
</tr>
<tr>
<td>(\ell &gt; 1)</td>
<td>8</td>
<td>Information lag</td>
</tr>
<tr>
<td>(\alpha \in [0, 1])</td>
<td>0.7307</td>
<td>Probability of not adjusting the price</td>
</tr>
<tr>
<td>(\beta \in (0, 1))</td>
<td>0.99</td>
<td>Subjective discount rate</td>
</tr>
<tr>
<td>(\gamma &gt; 0)</td>
<td>2</td>
<td>Relative risk aversion</td>
</tr>
<tr>
<td>(1/\eta &gt; 0)</td>
<td>4</td>
<td>Elasticity of labor supply</td>
</tr>
<tr>
<td>(\phi \in (0, 1])</td>
<td>(\frac{1}{2})</td>
<td>Labor returns to scale</td>
</tr>
<tr>
<td>(\theta &gt; 1)</td>
<td>11</td>
<td>Elasticity of substitution</td>
</tr>
<tr>
<td>(L)</td>
<td>1</td>
<td>Steady state time endowment</td>
</tr>
<tr>
<td>(\rho_\epsilon \in [0, 1])</td>
<td>.725</td>
<td>Autocorrelation of (\epsilon_t)</td>
</tr>
<tr>
<td>(\rho_u \in [0, 1])</td>
<td>.725</td>
<td>Autocorrelation of (u_t)</td>
</tr>
</tbody>
</table>

model’s recursive law of motion numerically because reasonable values of \(\ell\) induce a significant state vector. I calibrate parameters to quarterly data. The three parameters \(\alpha, \delta\) and \(\ell\) shape measurement bias in the model. I calibrate \(\alpha = 0.7307\) and \(\delta = 0.0797\). I set the lag \(\ell\) equal to 8 quarters. According to those numbers, the statistical authority samples on average \((1 - \delta)^\ell \approx 52\%\) of the universe of varieties [to be completed].

Let \(u(C) = (C^{1-\gamma} - 1)/(1 - \gamma)\) and \(h(L) = (L^{1+\eta} - 1)/(1 + \eta)\). The subjective discount rate \(\beta\) is set to 0.99 and household’s relative risk aversion \(\gamma\) equals 2. This value is smaller than the estimate in Rotemberg and Woodford (1999) but exceeds estimates in An and Schorfheide (2005), both for U.S. data. A value for \(\theta\) equal to 11 implies a markup over marginal costs of 10\%. The Frisch elasticity of labor supply to wages \(1/\eta\) is set to 4 and returns to scale in labor equal 1/2 roughly in compliance with the real business cycle literature. Under these choices \(\sigma\) equals 0.0498 which is consistent with recent estimates of this parameter in Altig, Christiano, Eichenbaum and Linde (2005) and Linde (2005) for U.S. data. Shocks follow AR(1) processes with AR-coefficient 0.725. The model does not feature inherent structural inertia beyond measurement bias whereas empirical applications usually lend support to positive lags in inflation and output gap equation. The shock processes take (partial) account of such empirical results. Table 2 summarizes parameter values.
5 Results on CPI bias and monetary policy

In this section, I describe the effects of CPI measurement bias. I illustrate CPI bias in the frequency domain and then demonstrate the extend and volatility of this bias in U.S. CPI inflation. I then discuss impulse response functions of measured and actual quantities.

5.1 Volatile bias in U.S. CPI inflation

The lag polynomial $a(L)$ can be understood as a particular filter that, once it is applied to actual inflation, recovers CPI inflation. Conversely, filtering CPI inflation with the inverse lag polynomial $a(L)^{-1}$ recovers actual inflation. Hence, knowledge of the filter and a time series of CPI inflation is sufficient to construct a time series of actual inflation. Under sum-ability of the coefficients of $a(L)$ the spectra of measured and actual inflation relate according to

$$S_{\pi_m}(\omega) = a(e^{-i\omega})a(e^{i\omega}) S_{\pi}(\omega).$$

Here $S_{\pi_m}$ ($S_\pi$) denotes the spectrum of measured (actual) inflation, $\omega$ displays a particular frequency and $i = \sqrt{-1}$. It is convenient to discuss the properties of the filter by discussing the properties of $a(.)a(.)$ because this discussion does not require any assumption regarding the inflation processes and hence is agnostic about $S_{\pi_m}$ ($S_\pi$).

Figure 2 plots the inverse of $a(.)a(.)$. The variance of actual inflation exceeds the variance of CPI inflation because the inverse filter is larger than unity across all frequencies. Furthermore, the filter downgrades low-frequency variation in CPI inflation. With quarterly data, horizons larger than roughly three years will be affected by the downgrade. As a result observed CPI inflation is less volatile and more persistent than actual inflation.

Figure 3 shows the bias $B$ when the filter is applied to U.S. CPI quarterly annualized inflation. The bias is sizeable, volatile and particular strong in periods of extreme values of CPI inflation. Recall the structural explanation of the bias: if inflation deviates strongly from steady state this indicates that prices of optimizing firms are very different from the weighted average of past prices. In such a situation underestimating the fraction of firms with recently optimized price produces a large bias. Absolute bias is on average roughly a tenth of measured inflation. Moreover, with autocorrelation 0.50 the bias exhibits considerable persistence. For comparison, autocorrelation of U.S. CPI inflation is 0.79. Measured inflation and bias are strongly negative correlated (−0.67). To confirm this strong negative correlation in theory, solve equation (10) for actual inflation and substitute for actual inflation in equation (9) to obtain $B_t = c(L)\pi^n_t$ where $c(L) = 1 - a(L)^{-1}$. Figure 4 depicts $c(L)$ – contemporaneously high CPI inflation goes with low bias.

5.2 Dynamic adjustment to shocks

Figures 5 and 6 show impulse response functions (IRFs) to positive shocks in $u_t$ and to negative shocks in $\epsilon_t$ because positive productivity shocks map inverted into $\epsilon_t$. In computing these IRFs it is assumed that the central bank refers to measured quantities when

\[\text{See Hamilton (1994), chapter 6.}\]
setting its interest rate according to

\[ \hat{i}_t = 1.5\pi_t^m + 0.5x_t^m. \]

In such a setup measurement bias is endogenous in the sense that it disturbs policy decisions and thereby economic outcomes.

Three important observations derive from figure 5 which shows IRFs to shocks in \( u_t \). First and most strikingly, measured quantities respond less strongly than actual quantities to the inflation shock. Second, differences between measured and actual variables are sizable for inflation and large for the output gap (the two bottom panels). Third, measurement bias is persistent for inflation and very persistent for the output gap with autocorrelations of 0.46 and 0.94, respectively.

The \( u_t \) shock induces prices of new varieties to exceed the actual price level. CPI inflation misses part of this price increase because the statistical authority can not record new varieties for a certain period of time. As a result, CPI inflation understates actual inflation. By the construction of real GDP data the inflation error translates into the opposite error in the output gap and measured real output and thereby the output gap is too high because measured inflation is too low. Measurement bias in inflation \( B_t \) is about a sixth of observed inflation in the initial period. After some quarters, because CPI inflation is more persistent and less volatile, CPI inflation exceeds actual inflation and the measurement bias overshoots. Measurement bias in the output gap is about a quarter of the observed initial output gap and peaks after three periods. The peak arises because the output gap
returns slowly back to steady state relative to inflation. As one would expect, the correlation between the two biases is negative (-0.20). When the information lag expires after 8 quarters this generates a visible adjustment in measurement biases.

The productivity shock in figure 6 induces a different pattern. Because inflation and the output gap move into the same direction, the measured output gap now reacts stronger than the actual output gap. Output gap bias exhibits similar persistence and peaks again after three quarters but now is of much smaller size. The reason for small bias is the moderate response of inflation to productivity shocks.

The quote in the introduction indicates a concern of policy makers that in times of strong productivity CPI inflation may overstate actual inflation. The inflation panel in figure 6 shows that this indeed is the prediction of the model. High productivity induces prices of new varieties to fall below the actual price level but CPI inflation misses part of this price decrease.

6 Conclusion

[To be added]
Figure 4: Coefficients of $a(L)$, $a(L)^{-1}$ and $c(L)$. Calibration is described in section 4.2
Figure 5: IRFs for positive shock in $u_t$ one standard deviation in size. The central bank responds to measured quantities. Calibration is as described in section 4.2.
Figure 6: IRFs for a positive productivity shock (negative shock in $\epsilon_t$) one standard deviation in size. The central bank responds to measured quantities. Calibration is as described in section 4.2.