Tonality as Implication-Realisation

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March 1999

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In Vos, P. & Leman, M. (Eds.),
Proceedings of Expert Meeting on Tonality Induction (pp. 121- 141)
Holland and Belgium, 6-9 April 1999

Abstract

The implication-realisation concept may be applied to various aspects of tonal perception and syntax. Consider the following four asymmetries in relationships between successive tones and chords. First, successive pure tones spanning an octave in the central pitch range sound more similar when the interval falls than when it rises. Second, for harmonic complex tones, rising octaves sound more similar than falling. Third, in mainstream tonal music, root progressions between major and/or minor triads (with one tone in common) more frequently span falling perfect fifths than rising perfect fifths. Fourth, in pairs of major/minor triads (with two tones in common), falling thirds between roots outnumber rising thirds. All these phenomena may traced to pitches that are implied (as fundamentals of incomplete harmonic series) but not physically present in the sounds concerned. The strongest of these implied pitches are: one octave below the main pitch of a pure tone; one octave above the main pitch of a harmonic complex tone; and at third and fifth intervals below the conventional roots of major and minor triads. Another kind of implication-realisation effect applies at perfect or authentic cadences. Both the prevalence distribution of chroma in a passage of music in a major or minor key, and the pitch-salience distribution of the tonic triad, correlate closely with the corresponding key profile. The prevalence distribution of chroma in the previous music may thus be regarded as an implication that is realised by the tonic triad, encapsulating the entire distribution in a single sonority.

Introduction

Meyer (1956) applied the idea of implication (or expectation) and realisation (or inhibition) to explain emotional responses to music: "Emotion or affect is aroused when a tendency to respond is arrested or inhibited" (p. 14). Inspired by Meyer, Narmour (1990) developed an implication-realisation model of melodic expectancy. Here, I invoke the implication-realisation concept to shed light on some aspects of western tonal-harmonic syntax that were not considered by Meyer or Narmour. My paper will close with a new interpretation of the tone profiles of Krumhansl and Kessler (1982), also based on the implication-realisation concept.

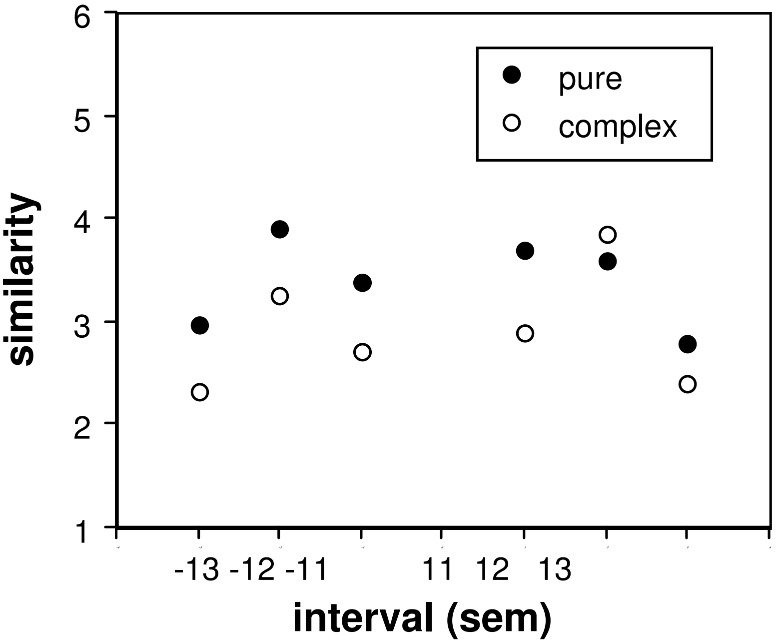


Figure 1. Mean similarity judgments of pure and complex (sawtooth) tones spanning rising and falling major seventh, octave, and minor ninth intervals. (Fundamental) frequencies ranged from 131 Hz (C3) to 523 Hz (C5). Participants comprised 9 musicians and 21 non-musicians. The experimental design included a wide range of both tonal and timbral intervals. The 95% confidence intervals of the means typically span ±0.5 units (min 0.4, max 0.6). From Parncutt (1989, Fig. 5.5 b).

Melody: Rising octaves

First, consider the tonal implications that may be generated by a single tone. Experiments on the similarity of successive pure and complex tones (Parncutt, 1989) revealed unexpected order effects: two successive pure tones separated by an octave were rated more similar if the octave fell than if it rose, whereas complex tones an octave apart were rated more similar if the octave interval rose rather than fell, by comparison to chromatically neighboring intervals (Figure 1).

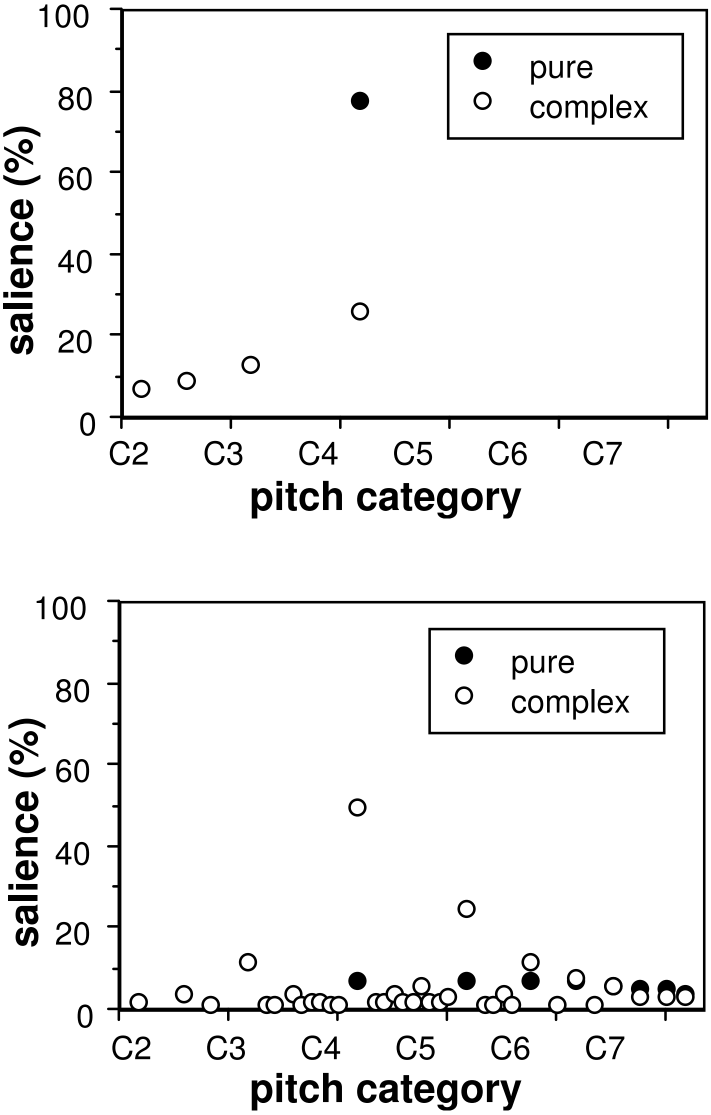


Figure 2. Calculated pitch salience profiles of a pure tone (upper panel) and a harmonic complex tone (lower panel) according to Parncutt's (1989) adaptation of the pitch model of Terhardt et al. (1982). Both tones are at D4 (294 Hz). Filled circles represent pure tone sensations (spectral pitches); open circles are complex tone sensations (virtual pitches). The strongest pitch of the pure tone corresponds to its frequency (pure tone sensation, or spectral pitch, at D4); the strongest subsidiary pitch lies an octave lower (complex tone sensation, or virtual pitch, at D3). The strongest pitch of the complex tone corresponds to its fundamental frequency (complex tone sensation, or virtual pitch, at D4); the strongest subsidiary pitch lies an octave higher (complex tone sensation, or virtual pitch, at D5).

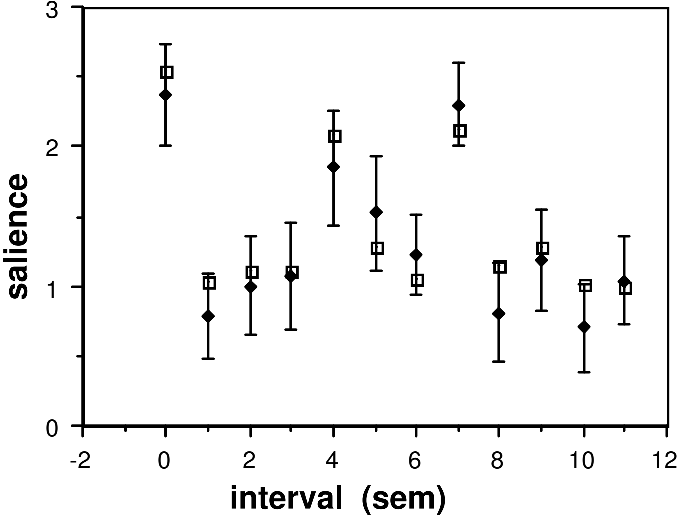
The findings could be accounted for as follows. Data of Houtgast (1976) suggest that the strongest subsidiary pitch of a pure tone lies an octave below its main pitch, while Terhardt, Stoll, Schermbach, & Parncutt (1986) observed in a pitch-matching experiment that the strongest subsidiary pitch of a harmonic complex tone in the central musical range (registers 3 and 4) lies an octave above the main pitch. Both these findings are consistent with predictions of Terhardt's (1972) theory of virtual pitch (Parncutt, 1989), as shown in Figure 2.

A related observation is that rising melodic octaves occur more frequently in music than falling octaves (Vos & Troost, 1989). But this effect is not necessarily due to subsidiary virtual pitches. Rising melodic intervals tend to be more common than falling for all intervals greater than about a perfect fourth. The following two effects combine to provide an explanation. First, a large (implicative) interval implies a smaller (realized) interval in the opposite direction (Narmour (1990). This tendency is explicable in terms of streaming (Bregman, 1990; Noorden, 1975): when a large leap threatens to break up a stream, coherence can best be restored by moving stepwise toward the mean pitch of the stream. Second, melodic phrases typically begin low, rise to a peak (producing tension), and fall again (relaxation) (Huron, 1996). A rising leap followed by a falling step fits this scheme better than a falling leap followed by a rising step.

Chord progressions: Falling fifths and thirds

The concept of implication-realisation may also explain certain assymmetries in chord progressions. The predominance of root progressions through falling (rather than rising) fifths and thirds in common-practice tonal music is well-known in music theory. To check this, Eberlein (1994, pp. 422-423) made a statistical analysis of chord progressions in a representative sample of scores composed between 1700 and 1850; composers were J. S. Bach, Handel, Mozart, Beethoven, and Mendelssohn. He confirmed that root progressions of a perfect fifth between root-position major or minor triads (regardless of their tonal function) more commonly fell (n=150 in his sample) than rose (n=45), and that root progressions of a third between root-position major and minor triads, where the triads have two tones in common, more commonly fell (n=26) than rose (n=3).

(a) major triad 047



(b) minor triad 037

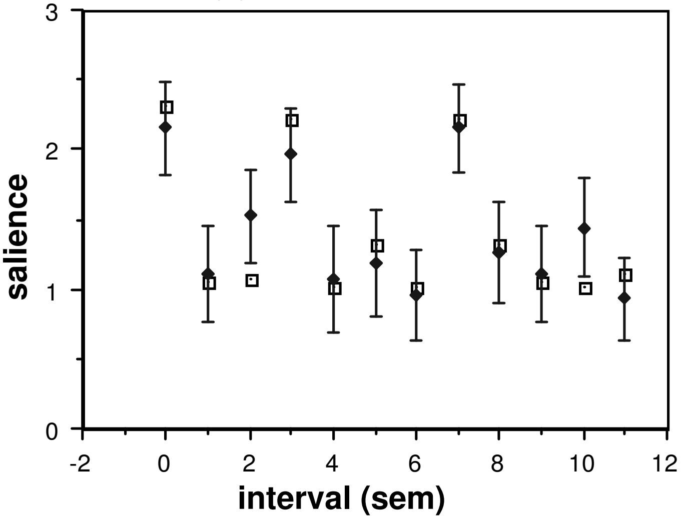


Figure 3. Experimental and calculated pitch-salience profiles of a major triad (upper panel) and a minor triad (lower panel) (from Parncutt, 1993, Fig. 2). Twenty-seven listeners (mainly musicians) rated how well a probe tone went with a preceding chord. Three other chords were tested in the same experiment: dom7, half-dim7 and dim7. Both chords and probe tones were constructed from octave-complex tones. Trials were rotated randomly around the chroma cycle. Points are mean experimental ratings, and bars are 95% confidence intervals. Squares are calculated values according to the model of Parncutt (1993), which is based on that of Parncutt (1988), but additionally accounts for masking.

Again, the implication-realisation concept can explain these observations. According to Parncutt's (1988) model of pitch salience in musical chords – which combines elements of the root algorithm of Terhardt (1982) and the pitch algorithm of Terhardt et al. (1982), and was tested against experimental data by Parncutt (1993) – the main subsidiary pitches (i.e., pitches not corresponding to notes) of major and minor triads lie at fifth and third intervals below the conventional root (where the subthird is minor in the case of the major triad, and major in the case of the minor triad). This prediction is consistent with data shown in Figure 3 (for each chord, compare results for perfect fourth with tritone, and major sixth with minor sixth). In falling-fifth and falling-third progressions, fifth and third intervals below the root of major and minor triads may thus be regarded as "implied" in the first chord and "realised" in the second.

			b^b	(–>)	B		
	a	–>	A				
	G	<–	g	–>	G	—	G
	f	–>	F				
	E				e	–>	E
			D	—	D		
	C				c	–>	C
C:	I		ii		V		I

Figure 4. Illustration of implication-realisation effects in a I - ii - V - I cadence in the key of C major. Tones actually present in the chords are shown in upper case. Implied pitches are in lower case. A forward implication is denoted by an arrow to the right; a backward implication by an arrow to the left.

A typical cadence might include several examples of such implication-realisation effects. Consider the progression I-ii-V-I illustrated in Figure 4. In the following, upper-case letters denote tones actually in a chord, and lower case denotes tones that are merely implied. The tones CEG in the first chord imply the tones f and a, both of which are realised in the second chord DFA. Only one of the two implied tones in the second chord (g) is realised in the third (there may, however, be a sense in which the other implied tone, b-flat, is "replaced" by the following B-natural). Both implied tones of the third chord (e and c) are realised in the final tonic triad.

Figure 4 suggests that realisation of implied tones in chords may be associated with a dynamic quality of moving forward (toward a cadence), by contrast to the static quality of progressions (prolongations?) that move in the opposite direction. Note also that this example of implication-realisation introduces an asymmetrical element into the "pitch commonality" (Parncutt, 1988, 1989, 1993) of pairs of chords, which – in contrast to effects investigated by Bharucha & Krumhansl (1983) – is independent of the context in which the chords occur.

An alternative explanation for the prevalence of falling fifth progressions in tonal music involves voice leading. Falling-fifth cadences evolved from late Medieval two-part cadences in which a major sixth interval resolved in contrary motion to an octave (Dahlhaus, 1967; Schenker, 1906). As the number of voices increased from two to three, the double-leading-note cadence emerged, which in turn was gradually replaced by the now-familiar falling fifth cadence (Eberlein, 1994). But an explanation based on medieval voice leading practices cannot, I would argue, account for the maintenance of the ?-V-I cadential formula during three centuries (17th, 18th, 19th) of far-reaching harmonic exploration and experimentation.

Modelling and interpreting Krumhansl's key profiles

As a final example of the implication-realisation concept in tonal music, consider Krumhansl's key profiles. To foreshadow my conclusion: I will argue that the key profiles arose historically from, and hence are essentially identical to, pitch-salience profiles of tonic triads. Moreover, since the frequency-of-occurrence distribution of chroma or pitch classes (hereafter, "prevalence distribution") of a passage of tonal music corresponds closely to the key profile, the tonic triad may be regarded as the realisation of an implication created by the prevalence profile of the preceding passage. To argue convincingly for this interpretation of the key profiles, however, I will first need to demonstrate that the pitch-salience profiles of major and minor triads indeed represent the best currently available model of the key profiles.

Krumhansl and Kessler (1982, p. 342) determined the relative stability of the twelve chromatic scale degrees in major and minor keys by asking listeners "how well, in a musical sense" octave-complex (Shepard) probe tones "fit into or went with" standard cadential progressions (IV-V-I, II-V-I, VI-V-I, and I alone). In part, their findings simply replicated what musicians and music theorists already knew and took for granted, although the rank order of importance of scale degrees had not previously been stated so explicitly. They concluded that the tonic is more stable than the other members of the tonic triad (the third and fifth degrees), followed by the fourth and sixth degrees, the second degree, the seventh (leading note), and finally the non-diatonic scale degrees; this pattern was much the same in both major and minor keys. A somewhat surprising (but similarly robust) finding was that the fifth scale degree is more stable than the third in major keys, but less stable in minor keys – contradicting received music-theoretic wisdom that the dominant scale degree is more stable than the mediant in both major and minor (cf., e.g., Lerdahl’s 1988 model of depth of embedding in tonal pitch space). While this finding may not apply to all tonal contexts that a music theorist would regard as clearly representing major or minor key, it is clearly valid in a normative or average sense.

So far, no quantitative model has been proposed that can satisfactorily account for the nature and origin of these profiles in a both a quantitative and qualitative sense – that is, combining high correlation coefficients with music-theoretically and psychologically convincing arguments.

Krumhansl and Kessler (1982) related the profiles to the structure of tonal music as follows: "In music certain tones are emphasized by their frequency of occurrence, particularly at phrase beginnings and endings, and these tones typically have longer duration and are given greater rhythmic stress” (p. 363). The effect of frequency of occurrence on tone profiles has more recently been demonstrated by Cuddy (1997), who measured tone profiles following non-diatonic sequences in which the frequency of occurrence of pitch classes had been carefully controlled; profiles from musically trained listeners could be accounted for by a combination of frequency of occurrence and learned key profiles (Oram & Cuddy, 1995). Conversely, Thompson and Cuddy (1997) found that tonal stability affects dynamics and timing in piano performance, such that for example tonally more stable notes may be held for longer durations than tonally less stable notes of the same notated duration; along similar lines, Sundberg (1988) included the profiles in his model of musical performance. Summing up these findings, it is apparent that surface cues in music performance influence tone profiles, and – conversely – tone profiles influence surface cues. This may be regarded as convergent evidence that the profiles arose from the patterns and regularities that obtain in tonal music itself.

But how did those patterns get there in the first place? Prevalence profiles in tonal music may be regarded as a result of a long historical series of changes in tonal syntax, which were presumably accompanied by corresponding changes in the way tonal music was perceived (Eberlein, 1994; Parncutt, 1996). Clearly, it would be impossible to understand such a development fully from a perceptual point of view – that would involve turning back the clock and subjecting listeners of past centuries to perceptual experiments. One could develop well-founded speculations based on the statistical properties of music that we know was regularly heard by a particular social group at a particular time, but that would be well beyond the present scope.

Another possibility is to explore the relationship between the profiles and relevant psychoacoustic measures, independent of stylistic or historic considerations.

Krumhansl (1990, Ch. 3) systematically compared the key profiles with well-known predictors for the consonance of musical intervals (Malmberg, 1918; Helmholtz, 1863; Hutchinson & Knopoff, 1978; Kameoka & Kuriyagawa, 1969). While many of these comparisons were quite successful – correlation coefficients between predictions of six models (each for major and minor keys) ranged from 0.38 to 0.83 (mean 0.61) – doubts may be raised regarding the conceptual validity of such an approach. From a music-theoretical or music-perceptual standpoint, there is no particularly compelling reason why the consonance of an isolated interval should be related to the stability of scale degrees in a tonal musical context. The observation, for example, that a harmonic interval of a perfect fifth is perceived as consonant by most western listeners, or treated as such by most western composers, cannot directly explain why, in a major key, the tonic is more stable than the dominant. The "smoothness" (i.e., lack of "roughness") of a perfect fifth is not directional; it does not apply to one tone of the interval more than it does to the other.

What aspect of the key profiles might consonance of intervals be able to explain? A possibility is that consonance is only responsible for the arrangement of intervals within the diatonic scale – not for the position of the tonic within the scale. No more than three different cyclic arrangements of 5 tones (T) and 2 semitones (S) exist within the octave (by "different", I mean sequences that cannot be mapped onto each other by transposition and/or inversion; see e.g. Rahn, 1980). The first of these is the familiar ordering TTSTTTS (or {1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1}), corresponding to the major scale and its inversions (TSTTTST, STTTSTT, etc.), and including the harmonic minor scale and medieval church modes. The second is the less familiar TSTTTTS, corresponding to the rising melodic minor scale. The third is the relatively rare TTTTTSS: a whole-tone scale with a chromatic passing tone. The reason why the first of these three cyclic arrangements is the most prevalent would appear to be simply that the first is the most consonant (Huron, 1994), and the main reason for that, in turn, may simply be that it contains the fewest tritones (one, rather than two in the second ordering, and three in the third). The dissonance of the tritone interval has played an important role in music theory since the Middle Ages (Randel, 1986). In pitch-class-theoretic terms, the tritone is the only interval class (or interval of 1 to 6 semitones, or unordered pc interval: Forte, 1977; Rahn, 1980) that is dissonant both when the tones sound simultaneously (harmonic interval) and when they sound successively (melodic interval). Harmonically, dissonance arises in the tritone dyad from two sources: sensory dissonance, or roughness due to an interaction between the third harmonic of the lower tone and the second of the upper; and root ambiguity, which prevents the interval from acquiring a tonal function in a tonal context (Terhardt, 1982; Parncutt, 1988). Melodically, dissonance arises from two entirely different sources. First, the interval is considerably larger than a step (1 or 2 semitones), and so relatively likely to cause a perceptual stream to segregate. Second, successive tones at an interval of a tritone have negligible pitch commonality, and so are unlikely to be perceived as harmonically related (Parncutt, 1989).

The question now arises as to whether variations in the consonance of intervals might explain why some modes of TTSTTTS are (or were) more common than others – or perhaps why, during the development of major-minor tonality in the 15th, 16th and 17th centuries, the Ionian and Aeolian modes finally emerged as "winners". A problem with any such explanation (including that of Huron, 1994), is that the consonance of certain intervals, and especially of the major and minor seconds, depends crucially upon whether they are played simultaneously (harmonically) or successively (melodically). Both the M2 and the m2 are normally regarded as harmonically dissonant (due to roughness) but melodically consonant (they are steps rather than leaps, and lie within the trill threshold of Miller & Heise, 1950). But the music of the 15th, 16th and 17th centuries was of course both harmonic and contrapuntal; that is, both simultaneous and successive tone relationships played an important role. Since the seven modes of TTSTTTS differ only in the ordering of tones and semitones, and whole-tone intervals are sometimes more (harmonically) and sometimes less (melodically) consonant than semitones, explanations based on consonance cannot explain preferences for one mode over another.

Table 1. Comparison of the diatonic scale with pitch-salience profiles of chords built on different degrees of the scale. The top row gives the (conventional) root of the triad under consideration. The table contains Pearson correlation coefficients between the pitch-salience profile of the triad according to Parncutt (1988) and the diatonic scale represented as the array {1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1}. Correlations exceeding 0.5 are bold.

root	C	D	E	F	G	A	B
major triad	**.57**	.37	.14	.44	**.61**	.24	-.13
minor triad	.11	.47	**.64**	-.16	.04	**.67**	.31

How, then, can the position of the tonic(s) in the major/minor system be explained? A possibility might be to regard the tonic of a major/minor key as a triad, and to compare the pitch-salience profiles of major and minor triads with each of the different modes of the TTSTTTS scale in turn. The results of such a comparison are shown in Table 1.

Unfortunately, this procedure also turns out to be unsuccessful: it cannot show why C and A are the preferred tonics of the scale CDEFGAB in the major/minor system. Instead, the table incorrectly suggests that C and G compete for the major tonic, and E and A for the minor tonic. Clearly, it does not work to assume that the diatonic scale preexisted, and that the tonic triad was simply the triad that best matched the scale. As explained above, the problem is solved if we instead take typical prevalence profiles of tonal music as our starting point. However, we are again left with the question of whether those profiles came from. I will return to this question again below.

Table 2. Comparison of key profiles with four predictors. The key profiles of Krumhansl and Kessler (1982) (labeled "K&K82"; as reported in Krumhansl, 1990, p. 30) are compared with stimulus profile ("stimulus": number of times each notated pitch occurs in the chord progressions used by K&K82, see text); calculated pitch salience ("Pmo88": model of Parncutt 1988 with root-support weights P1/P8 = 10, P5 = 5, M3 = 3, m7 = 2, M2/M9 = 1; the values shown are called pitch weights in the published model); weighted sum of pitch-weight profiles for the chords used by K&K82 ("cadence": based on Pmo88); and calculated pitch weight according to a model that additionally accounts for mutual masking among tones at the input ("Pmo93": model of Parncutt 1993 with root-support weights as above, plus k_M = 6 dB/octave and k_W = 1).

(a) Major key

	C		D		E	F		G		A		B
K&K82	6.35	2.23	3.48	2.33	4.38	4.09	2.52	5.19	2.39	3.66	2.29	2.88
stimulus	8	0	4	0	7	2	0	9	0	3	0	3
Pmo88	18	0	3	3	10	6	2	10	3	7	1	0
cadence	150	12	78	29	93	78	14	126	24	86	30	36
Pmo93	12.6	0	2.1	2	6.2	4.4	1.2	6.6	2.2	4.4	0.7	0

Pearson correlation coefficients

	K&K82	stimulus	Pmo88	cadence
stimulus	0.914			
Pmo88	0.943	0.816		
cadence	0.975	0.920	0.916	
Pmo93	0.944	0.797	0.997	0.912

(b) Minor key

	C		D	Eb		F		G	Ab			B
K&K82	6.33	2.68	3.52	5.38	2.6	3.53	2.54	4.75	3.98	2.69	3.34	3.17
stimulus	8	0	4	7	0	2	0	9	3	0	0	3

Pmo88	15	1	2	13	0	8	0	10	8	2	1	3
cadence	129	34	56	99	32	83	3	123	89	21	36	51
Pmo93	10.3	0.6	1.38.4	0	5.3	0	7.4	5.1	1.5	0.7	1.9	

Pearson correlation coefficients

	K&K82	stimulus	Pmo88	cadence
stimulus	0.900			
Pmo88	0.947	0.862		
cadence	0.910	0.909	0.935	
Pmo93	0.949	0.878	0.997	0.944

(c) Regression model of K&K82 profiles across 24 values (major + minor)

Predictor	Coef	Stdev	t-ratio	p
constant	2.23	0.17	13.4	0.000
stimulus	0.103	0.054	1.90	0.072
Pmo88	0.117	0.037	3.22	0.004
cadence	0.0070	0.0060	1.17	0.254

Given that the consonance of intervals cannot explain Krumhansl's profiles, let us now explore some other options. Perhaps the simplest model for Krumhansl’s key profiles is the tonic triad, which may be expressed relative to the chromatic scale as {1 0 0 0 1 0 0 1 0 0 0 0} – but of course this cannot account for variations in the tonal stability of tones both, within and outside the triad. A somewhat more sophisticated and accurate model involves counting the number of times each notated pitch occurred in the chord progressions originally used to determine the profiles (Butler, 1989). Following Butler, I refer to this model as the stimulus profile. Krumhansl and Kessler’s C-major profiles were averaged over four stimuli: three progressions (F-G-C, d-G-C, and a-G-C, where upper case denotes major triad and lower case denotes minor) and a single triad (C). Replacing the single triad by a progression (C C C) to give it the same weight as the other progressions, the stimulus profile underlying the C-major key profile may be obtained by counting how many times each chroma occurs in the combined chord sequence F G C d G C a G C C C C (cf. Parncutt, 1989, pp. 159–160).

Table 2 compares stimulus profiles calculated in this way with the corresponding standard key profiles. The resultant correlation coefficients are 0.91 for the major key and 0.90 for the minor. Thus, as Butler (1989) pointed out (although note that he calculated the stimulus profiles slightly differently), most of the information contained the key profiles is already contained in the notation of typical cadential progressions, in particular in the progressions used in Krumhansl and Kessler's experiments.

Alternatively, the origin of the profiles may be explained by again invoking the concept of pitch salience. Parncutt (1989, 1994) assumed that pitch-salience profiles calculated for individual, isolated chords may be added across time to obtain the tone profile for a chord progression. The model neglected any perceptual interactions that may occur between chords, or the attraction of a listener's attention to melodic streams (voices) within the progression (cf. Bregman, 1990). Results were quantitatively more promising than those obtained by Butler (1989) and Krumhansl (1990), with a correlation coefficient of 0.98 for the major key and 0.94 for the minor. Later, Parncutt (1994) developed a simpler, octave-generalized model, and compared the individual profiles of Krumhansl and Kessler (1982) following their 8 different stimulus presentations (3 cadential progressions and a single tonic triad, each in both major and minor mode); correlation coefficients ranged from 0.81 to 0.98, with a mean of 0.92. Similarly, Huron and Parncutt (1993), assuming that the last chord of a sequence contributes more to the composite profile than previous chords (recency effect) and that sensory memory decays exponentially with a half-life of roughly one second, accurately modeled gradual changes in tone profiles by Krumhansl and Kessler (1982) during modulating chord progressions.

Krumhansl and Kessler had, of course, already found that the profile produced by a single tonic triad correlated significantly with the profile produced by a cadential progression in the same key. This suggests a simple solution: key profiles correspond to the pitch-salience profiles of their tonic triads. As shown in Table 2, correlations between the standard key profiles and the corresponding pitch-salience profiles calculated by Parncutt (1988) (labeled "Pmo88") are better than correlations with stimulus profiles after Butler: 0.94 for the major key and 0.95 for the minor. Interpreting this result, it is not surprising that a pitch "evoked" by a chord (according to Terhardt's pitch model) also "fits into or goes with" the chord of its tonality in a musical sense (as Krumhansl and Kessler measured). The reason why the first and second chords of a three-chord cadence have so little effect on the key profile may be that the subdominant and dominant chords create a strong expectation that the tonic will follow – a classic case of implication-realisation. So, in the end, all that matters for the profile is the tonic triad. The degree to which any tone may be heard to complete a tonal passage (i.e., a probe-tone rating) would appear to depend simply on how well that tone represents or stands for the tonic triad (i.e., the salience of the tone's pitch within the tonic triad).

An explanation of the profiles based on a single chord is preferable to that proposed in Parncutt (1989), for two reasons. The first is quantitative: an explanation based on a single chord is just as powerful (at least in the case of the minor key), but mathematically considerably simpler and less arbitrary, than an explanation based on a chord progression. Second, an explanation based on a single chordd may more easily be interpreted – first, as just described (a pitch "evoked" by a chord should also "fit into or goes with" it), and second, in terms of implication and realisation (see below).

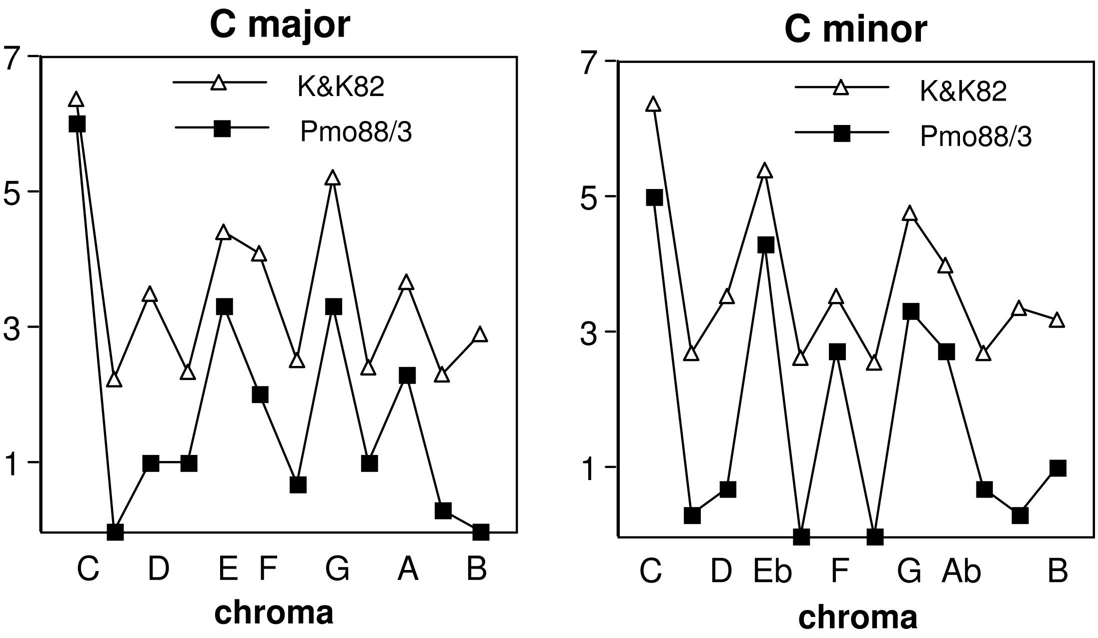


Figure 4. Comparison of major and minor key profiles of Krumhansl & Kessler (1982) (open triangles) with calculated pitch salience within the tonic triad (Parncutt, 1988 with root-support weights P1/P8 = 10, P5 = 5, M3 = 3, m7 = 2, M2/M9 = 1, m3 = 0) (filled squares). The vertical axis represents both mean experimental goodness-of-fit rating on a 7-point scale (for Krumhansl) and calculated pitch-class weight, divided by 3 for ease of comparison (for Parncutt).

Figure 4 illustrates the quantitative relationship between Krumhansl's key profiles (here drawn relative to an arbitrary tonic of C) and profiles calculated for the corresponding triads. From visual inspection, it is apparent that the main structural features of the profiles are reasonably well reflected by the model, including for example the relative stability of the third and fifth scale degrees in major and minor keys. The model deviates from the data in the following ways: in the major key, E^b is wrongly predicted to be about equally as salient as D, E to be as salient as G, and B to be less salient than B^b; and in the minor key, F is wrongly predicted to be more salient than D, and B more salient than B^b. It might be possible to explain these occasional deviations in terms of additional factors such as perception of roughness and voice leading, but to be plausible, such explanations would need to be applied systematically to all scale steps in both modes, and would ultimately involve extending the model. Such an investigation would be beyond the present scope. It suffices for the present to claim that, apart from some small deviations, the calculated profiles fit the key profiles remarkably well. This may be interpreted as confirmation of the music-theoretic idea (e.g., Riemann, 1877) that the tonic of a major or minor key is in the first place a triad, not a single tone; it is the triad that acts as the reference to which all tones in a major-minor tonality relate.

Krumhansl (1990) observed that the prevalence distribution of a tonal passage correlates strongly with the key profile (cf. Cuddy, 1997) . Similarly, Huron (1993) observed that more stable pitches in the profile are generally more likely to be doubled than less stable pitches (e.g., in four-part voicings of triads). Observations such as these may now be seen in a new light. According to the above arguments, the prevalence distribution of a tonal passage corresponds to the pitch-salience profile of the tonic triad – consistent, incidentally, with Schenker's (1906) insight that a tonal work is a temporal unfolding or prolongation of the tonic triad. Thus, a major or minor triad may be implied in tonal music by a musical passage whose prevalence distribution corresponds to the chord's pitch-salience profile (but during which the triad in root position may be deliberately avoided), and then finally realised at the cadence. The emotional connotations of an implication-realisation effect of this kind may underlie the feeling of tonal closure that follows the "long, firm, and unequivocally resolved section in the tonic at the end, dramatic if need be, but clearly reducing all the harmonic tensions of the [classical] work" (Rosen, 1972, p. 75).

In making a strong connection between the perception of a single chord and the perception of a tonality, I am not suggesting that an isolated chord will normally be perceived as a tonic. On the contrary, an isolated major or minor triad could be the tonic, the dominant, or some other function; or it could have no tonal function at all. There is no reason, for example, why the cognitive structure of a musical key should be instantiated by the presentation a single, isolated chord followed by a probe tone in an experimental trial. In tonal music, to establish a key requires several chords; classical composers, for example, generally devote a good deal of time to establishing the main tonality at the start of a piece (Rosen, 1972). (If, of course, a listener expects the first triad of a piece to be the tonic, which it almost always is in some musical styles – e.g., Bach's WTC Preludes – s/he may perceive tonality in a single triad; see Cohen, 1991.)

The connection between the pitch-salience profiles of triads and the key profiles may best be understood from a historical perspective. Major and minor triads first began to function as harmonic units during the late Middle Ages and Renaissance (14th - 16th centuries). The musical usefulness, and hence prevalence, of these triads in western music is not difficult to explain: they are the only possible simultaneities in the chromatic scale that contain a perfect fifth (producing a clear root, and hence a clear tonal function) and no major or minor seconds (hence low dissonance) (Parncutt, 1988). The historical point at which tonality, and hence key profiles, "emerged" depends on how tonality is defined (harmonic tonality, major-minor tonality, etc.); estimates range from the 15th to the 17th centuries (Dahlhaus, 1967; Eberlein, 1994; Randel, 1971). The strong mathematical relationship between the key profiles and the pitch-salience profiles of tonic triads invites speculation regarding the perceptual-historical process by which the key profiles emerged. As major and minor triads became commonplace, listeners, performers and composers may first have internalized their pitch-salience profiles. As major and minor triads increasingly took on the function of tonics (points of reference and of closure) in musical forms and structures, composers may gradually and intuitively have changed the prevalence distributions of their music to match the pitch-salience profiles of the corresponding tonic triads. (This, by the way, is a hypothesis that could be tested by statistical analysis of representative samples of music from the periods in question.) The frequency of occurrence of specific harmonic and contrapuntal progressions would have been adjusted by trial and error, taking into account other constraints including both existing compositional rules such as those for voice-leading, and by intuitive or perceptual preferences, such as for rising-fourth and falling-third progressions between successive chords. As the correspondence between prevalence distributions and tonic pitch salience profiles strengthened, so too did the feeling of key, the perceived strength and clarity of the tonal organisation, the feeling of resolution or closure at cadences, and of course the clarity of the cognitive schemas reflected by Krumhansl's key profiles.

A modern consequence of this process is that, in spite of a century musical modernism, the most familiar musical sonorities, to western ears, are still the major and minor triads. Thompson and Parncutt (1977) measured tone profiles of various sonorities, including selected simultaneous dyads (perfect fifths, major thirds). The profiles they obtained included some peaks that could not be accounted for by Terhardt's pitch theory. Instead, they corresponded to missing elements of major and minor triads, suggesting that listeners perceived these stimuli as incomplete versions of the tonal sonorities with which they were most familiar.

Returning to the question posed above of why the major and minor scales have the tonics that they do, we may now be in a position to advance a plausible, but necessarily speculative, historical explanation, in three stages. The first stage dates back at least as far as ancient Greece; it involves the development of a general preference for the diatonic scale (TTSTTTS, and rotations thereof), because it is the least dissonant scale that enables conjunct movement to and from all scale degrees (i.e., with no intervals greater than 2 semitones between adjacent scale steps). The second stage corresponds to the 14th, 15th and 16th centuries (late Middle Ages and Renaissance), and involved the gradual establishment of major and minor triads as the most important musical sonorities. The third stage overlapped with the second, beginning in the 15th century and extending into the 17th. During this time, prevalence profiles were intuitively adjusted to correspond to the by-now-familiar pitch-salience profiles of major and minor triads. The result was the emergence of the major and minor scales – that is, the narrowing down of the various possible tonics of the diatonic scale to just two.

Conclusion

Bringing these threads together, it would appear that, in a passage of major-minor music leading to an authentic cadence, at least four implication-realisation effects occur simultaneously. First, the prevalence distribution of tones in the preceding music is realised as the pitch-salience profile of the tonic chord. Second, the root of the tonic chord is implied by the previous dominant chord – in fact, in chains of falling fifths and thirds such as IV II V I, each chord implies the next. Two other well-known effects of implication-realisation may be added to this list. Third, the tonally unstable leading tone resolves to its nearest neighbour. Fourth, the seventh of the dominant, which is dissonant relative to that chord, resolves in the opposite direction.

I sum up with two somewhat speculative generalisations (hypotheses?) that are consistent with the above observations and arguments. First, the emotional connotations of tonal cadences (e.g., a listener's response to the appearance or non-appearance of an expected tonic) would appear to be well described by Meyer's (1956) theory of emotion and meaning, at several simultaneous levels of implication-realisation. This invites empirical investigation: do listeners experience different emotional responses when certain implications are realised and others not, in different combinations? Second, the (Viennese) classical style may be characterized by the generation, through a combination of tone distributions, voice-leading, motivic development and so on, of clear implications that are eventually realised according to consistent and predictable procedures, resulting in a "classical" sense of balance.

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