Tonal implications of harmonic and melodic Tn-sets

Richard Parncutt, University of Graz

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Schönberg's term "pantonal" (Reti, 1958) implies that, strictly speaking, music composed of tones can never be completely atonal. With the exception of the null set (cardinality = 0) and the entire chromatic aggregate (cardinality = 12), all pitch-class (pc) sets have tonal implications that depend on the intervals between the tones in the set. As a rule, the "atonal" repertoire tries to avoid tonal references by favoring pc-sets with relatively weak tonal implications (a well-known exception is Berg's violin concerto, a work that is usually regarded as serial but not atonal, because for example the row at the beginning of the first movement begins with a minor triad). It follows that "atonal" composers deliberately seek out tonally weak or ambiguous pc-sets for use in their compositions, and that they find such sets either in the existing "atonal" literature or by aurally guided exploration. The aim of this paper is to present a new method by which composers can systematically seek and find pc-sets of any specified cardinality and strength of tonal implication.

The tonal implications of a musical fragment depend not only on the underlying pc-set but also on its musical realization (octave register, doubling, durations, loudness, timbres, synchronicity, order…). A tone is more likely to be perceived as a tonal center if it is doubled (appears in more than one octave), is repeated at different times, has a long duration or is simply louder than other tones (Oram & Cuddy, 1995; Parncutt, 1988, 1997). Here, I assume – problematically, but also necessarily in an analytical approach – that it is possible to separate (i) the effect of the intervals within a set on the set's tonal implications from (ii) the effect of the set's specific realization or voicing, and focus only on (i).

According to Rahn (1980), a *Tn-set* is a pc-set that is invariant under transposition but not inversion. For example, set 3-11 is asymmetrical (its inversion within the octave cannot be mapped onto the original set by transposition); it therefore comprises two Tn-sets, 3-11A (037, the minor triad) and 3-11B (047, the major triad). I define the *perceptual profile* of a Tn-set as a set of 12 weights, one for each of the 12 chromatic scale degrees. Each weight reflects the *perceptual salience* of that scale degree. I further define two kinds of perceptual profile, *harmonic* and *tonal*, and present in the following paragraphs algorithms for calculating these profiles that are based on existing empirical data and perceptual-cognitive theory.

The *harmonic profile* of a Tn-set comprises estimates of the probability that each chromatic pitch will be perceived as the *root* of the set when it is presented harmonically (i.e., as a sonority). In Parncutt (1988), I developed a simple algorithm for pc-salience within harmonically presented pc-sets that was based on the virtual pitch algorithm of Terhardt et al. (1982) and the chord-root model of Terhardt (1982). The theory assumes the root of a chord is a virtual pitch generated by the chord's tones. The intervals octave/unison, perfect fifth, major third, minor seventh and major second/ninth are assumed to "support" the root (they are *root-support intervals*) because they are octave generalizations of the intervals between spectral and virtual pitches in typical harmonic complex tones such as voiced speech sounds (i.e. between harmonic overtones and the fundamental). The chord-root model includes free parameters called *root-support weights,* which are set in the present paper to the following values: octave/unison, 10; fifth, 5; major third, 3; minor seventh, 2; major second/ninth, 1; all other intervals including the minor third, zero. The predictions of the chord-root algorithm (including an additional masking procedure) were tested experimentally in Parncutt (1993) for a limited set of chords of octave-complex (Shepard) tones; Krumhansl and Kessler (1982) obtained essentially the same data when they asked how well octave-complex probe tones follow single chords (rather than short progressions). The predictions of the algorithm may also be considered to apply typical or average voicing of a given Tn-set realized as regular musical tones (harmonic complex tones).

The *tonal profile* of a Tn-set comprises estimates of the probability that each chromatic scale degree will be perceived as the *tonic* when the Tn-set is realized melodically (successively) in random order(s) and register(s). The major and minor key profiles of Krumhansl and Kessler (1982) comprise 24 values that may be regarded as measures of the stability of chromatic scale steps in the context of major and minor keys (12 for each). I adapt these profiles for the present purpose by subtracting a constant (2.23) from all values so that the minimum value becomes zero. I then estimate the probability that a given Tn-set type will occur in a given key by adding up the stability, according to Krumhansl's profiles, of the notes of that set in that key. For example, the probability that the set CEF# will occur in the key of C major is estimated by adding up the stability of C, of E and of F# in the C major key profile. Finally, I calculate the tonal profile of the Tn-set as a weighted mean of all 24 K-K profiles (one for each major and minor key), where the weights are the probabilities calculated in the previous step (i.e. how often we expect the Tn-set in question to occur in each key). Although the algorithm is limited by the assumption that tonality is limited to the Western major and minor modes, it yields intuitively reasonable results, as shown in the appendix.

The perceptual (harmonic and tonal) profiles for the C major and minor triads according to these procedures, normalized so that their mean is 10 and rounded to the nearest whole number, are:

pitch class	**in semitones**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
	as letter	**C**		**D**		**E**	**F**		**G**		**G**		**B**
major triad	harmonic	34	0	6	6	19	11	4	19	6	13	2	0
3-11B (047)	tonal	22	0	13	5	17	10	0	22	4	13	4	9
minor triad	harmonic	29	2	4	25	0	15	0	19	15	4	2	6
3-11A (037)	tonal	14	7	10	12	8	11	7	14	10	8	11	8

The correlation coefficient between the harmonic and tonal profiles is quite high ($r = 0.84$ for both major and minor), and the results for major and minor are also similar: in both harmonic profiles, the most likely root is the conventional root, the third and fifth have relatively high salience, and the fourth and sixth are strongly implied. The approximately equal stability of root and fifth in the tonal profiles is difficult to interpret and suggests that the theory may be incomplete – the problem may be that it does not explicitly include the tonicizing property of leading tones.

The perceptual profile of a Tn-set of cardinality between 1 and 11 always has peaks, which means that it is always to some extent tonal: the clearer the peaks, the clearer the tonality. In Parncutt (1988) I developed a simple mathematical formulation of the "peakedness" of a pc-set's perceptual profile and called it *root ambiguity*. It was calculated by dividing the sum of the 12 values by their maximum and taking the square root of the result; the square root came from a model developed to account for empirical data on the number of tones simultaneously perceived in a set of musical and non-musical sonorities (their *multiplicity*) in Parncutt (1989). According to this procedure, the calculated harmonic ambiguity of the major triad is 1.87, which makes it the least ambiguous of all 19 Tn-sets of cardinality 3 and is consistent with its ability to blend (to fuse perceptually).

The appendix presents the calculated perceptual profiles of all Tn-sets of cardinality 3. These data have interesting compositional and music-analytical applications, but it is important also to consider the *roughness* of the corresponding sonorities. In a first approximation, the roughest interval is the minor second, followed by the major second and tritone (Plomp & Levelt, 1965; Huron, 1994). Consider first the calculated ambiguity of the *harmonic profiles*. The least ambiguous sets are predicted to be 047 (major), 035 (part of a seventh chord), 027 (suspended), and 037 (minor), in that order. The reason why 037 is more prevalent than 027 or 035 in tonal music evidently involves the roughness of the major second interval within 035 and 027. The most ambiguous Tn-sets of cardinality 3 are predicted to be 036, followed by 012, 013 and 023, then by 014, 034, 046 and 048. The model predicts that 036 has four root candidates of roughly equal salience, making it highly ambiguous: none of its three tones is reinforced by a root-support interval, so all have equal salience, and a non-chord tone - 8 relative to 036, or Ab relative to CEbGb - is reinforced by all three tones, which gives it the character of a "pitch at the missing fundamental". The prevalence of 036 in tonal music may be due to the following factors: it is relatively smooth because it contains no major or minor seconds; it is a subset of the prevalent major-minor (dominant) seventh chord (4-27B or 0368; the least ambiguous Tn-set of cardinality 4); and it is a subset of the standard major and minor scale sets (Parncutt, 2006). The other listed sets are less prevalent because they contain second intervals. These sets may therefore be considered suitable for composition of "atonal" music in three parts. Of course, the situation is more complex than this because the ambiguity of a set also depends on the context in which it appears.

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Appendix: Calculated perceptual profiles of all Tn-sets of cardinality 3

Legend

Tn-set labels	harmonic profile	ambiguity of harmonic profile
correlation between profiles	tonal profile	ambiguity of tonal profile

3- 1 (012)	21	19	23	4	4	10	10	10	6	6	8	2	a=2.29
r = 0.72	11	10	11	9	10	11	9	11	9	11	10	9	a=3.26
3- 2A (013)	19	21	4	23	0	13	10	0	15	6	2	8	a=2.29
r = 0.75	11	11	8	12	8	11	9	9	12	8	11	8	a=3.11
3- 2B (023)	21	2	23	19	4	13	0	10	15	0	8	6	a=2.29
r = 0.75	12	8	11	11	8	11	8	12	9	9	11	8	a=3.11
3- 3A (014)	25	19	6	4	19	10	13	0	6	15	2	2	a=2.20
r = 0.75	11	11	9	9	12	10	9	9	11	11	8	10	a=3.13
3- 3B (034)	25	2	6	19	19	13	4	0	15	10	2	6	a=2.20
r = 0.75	12	9	9	11	11	10	8	11	11	9	9	10	a=3.13
3- 4A (015)	19	25	4	6	0	29	10	4	6	6	11	2	a=2.05
r = 0.83	13	11	9	10	8	14	8	9	11	9	11	7	a=2.97
3- 4B (045)	25	6	6	2	19	29	4	4	6	10	11	0	a=2.05
r = 0.83	14	8	10	9	11	13	7	11	9	11	9	8	a=2.97
3- 5A (016)	19	19	10	4	2	10	29	0	10	6	2	11	a=2.05
r = 0.73	10	13	8	10	10	10	12	8	11	10	10	10	a=3.08
3- 5B (056)	19	6	10	2	2	29	19	4	10	0	11	10	a=2.05
r = 0.73	12	10	10	10	8	13	10	10	10	10	11	8	a=3.08
3- 6 (024)	27	0	25	0	23	10	4	10	6	10	8	0	a=2.12
r = 0.75	12	8	12	8	12	10	8	12	8	12	8	10	a=3.11
3- 7A (025)	21	6	23	2	4	29	0	13	6	0	17	0	a=2.05
r = 0.85	14	7	12	9	8	14	7	12	8	11	11	7	a=2.95
3- 7B (035)	19	8	4	21	0	32	0	4	15	0	11	6	a=1.93
r = 0.85	14	8	9	12	7	14	7	11	11	8	12	7	a=2.95
3- 8A (026)	21	0	29	0	6	10	19	10	10	0	8	10	a=2.05
r = 0.59	11	9	12	9	9	10	11	11	8	11	9	10	a=3.14
3- 8B (046)	25	0	11	0	21	10	23	0	10	10	2	10	a=2.20
r = 0.59	11	10	9	9	12	9	11	10	9	11	8	11	a=3.14
3- 9 (027)	30	0	23	6	4	11	0	29	6	4	8	0	a=1.98
r = 0.90	14	6	14	8	10	11	7	15	7	11	10	8	a=2.82
3-10 (036)	19	2	10	19	2	13	19	0	19	0	2	15	a=2.51
r = 0.62	11	10	8	12	8	10	11	10	11	8	11	10	a=3.11
3-11A (037)	29	2	4	25	0	15	0	19	15	4	2	6	a=2.05
r = 0.84	14	7	10	12	8	11	7	14	10	8	11	8	a=2.95
3-11B (047)	34	0	6	6	19	11	4	19	6	13	2	0	a=1.87
r = 0.84	14	7	11	8	12	10	7	14	8	11	8	10	a=2.95
3-12 (048)	25	10	6	0	25	10	6	0	25	10	6	0	a=2.20
r = 0.74	12	10	8	10	12	10	8	10	12	10	8	10	a=3.15