# 'Endogeneous' Growth Models and the 'Classical' Tradition

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> Profits do not necessarily fall with the increase of the quantity of capital because the demand for capital is infinite and is governed by the same law as population itself. They are both checked by the rise in the price of food, and the consequent increase in the price of labour. If there were no such rise, what could prevent population and capital from increasing without limit? (Ricardo, *Works*, VI, p. 301).

## I. Introduction

Growth theory, like several other subjects in economics, had remarkable ups and downs in the history of our subject. A major focus of attention of the classical economists from Adam Smith to David Ricardo, and then of Karl Marx, who sought to revive the classical tradition purged of what he considered to be false abstractions and misconceptions, the problem of economic growth was almost completely lost sight of at the time of the so-called 'marginal revolution', championed by William Stanley Jevons, Carl Menger and Léon Walras. While there were notable exceptions to the rule, around the turn of the century economists were predominantly concerned with the problem of value. The interest in the problem of economic growth was re-ignated by a contribution of John von Neumann in the 1930s. However, a greater direct impact on the profession as a whole had the attempts to generalise Keynes's principle of effective demand to the long run. It was particularly Roy Harrod's 1937 contribution that gave rise to a large literature devoted to the study of economic growth and business cycles. The 'Instability Principle' enunciated by Harrod with regard to the process of capital accumulation was countered, in the mid 1950s, by the neoclassical economists Trevor Swan and Robert Solow, who showed that on the basis of sufficiently strong neoclassical assumptions the economic system would gravitate towards a steady state, with the rate of expansion equal to some exogenously given 'natural' rate of growth. In these models Say's law was assumed to hold, implying the full employment of labour and full capacity utilization. At the same time Nicholas Kaldor put forward the postkeynesian model of growth and distribution, which started also from the assumption of full employment and full capacity utilization. With the rate of growth of the labour supply and the rate of growth of labour productivity given from outside on one hand and the growth rate of investment given by the 'animal spitis of the investors' on the other, both the neoclassical and the post-Keynesian theory ascertained, via different routes, the distribution of income between wages and profits compatible with the given long-term growth rate. While the former assumed a given overall saving rate and a flexible capital-output ratio (via changing proportions of capital and labour by means of which a unit of social output could be produced), the latter assumed *prima facie* a flexible overall saving rate (via a changing distribution of income and different propensities to save out of wages and profits) and a fixed capital-output ratio.

The 1960s could be seen as the 'golden age' of Solovian growth economics: it brought a host of theoretical and empirical studies. However, in the 1970s and early 1980s growth economics as a whole was marginalised. The situation changed dramatically in the mid 1980s, when growth economics started to boom again, following the lead of Paul Romer and Robert Lucas. A formidable industry of theoretical and empirical research on economic research shot up like a mushroom. Also described as 'new' growth theory (NGT) to indicate the claim to originality, some advocates are quite explicit in their view that NGT will revolutionize the way economists think about certain problems (see Grossman and Helpman, 1994, p. 42). The emphasis is on 'endogenous' mechanisms generating economic growth, that is, long-term growth is determined 'within the model, rather than by some exogenously growing variables like unexplained technological progress' (Barro and

Sala-i-Martin, 1995, p. 38). This is considered the main distinguishing feature between NGT and old, Solovian, growth theory. Other characteristic features of NGT are said to be the incorporation of economies of scale in the model and of providing a solid microfoundation of saving (alias investment) behaviour.

In this chapter an attempt will be made to relate some of the most prominent models of the NGT literature to the 'classical' tradition of economic thought. It will indeed be argued that in a very precise sense the NGT can be said to involve a *return* to modes of thought and the method of analysis characteristic of the classical authors. In terms of method, the NGT is *long-period* theory, advocated by Adam Smith and developed by David Ricardo. In terms of content, many of the models of the 'new' growth theory (NGMs) dispense with the traditional neoclassical determination of the rate of profit in terms of the supply of and demand for 'capital'. The following discussion attempts to clarify this fact.

Scrutiny shows that the contributions to the theory of value and distribution of 'classical' derivation, notwithstanding the many differences between different authors, share a common feature: in investigating the relationship between the system of relative prices and income distribution they start from the same set of *data.* These data concern

(i) the technical conditions of production of the various commodities;
(ii) the size and composition of the social product;
(iii) one of the distributive variables: either the ruling wage rate(s) or the ruling rate of profit; and
(iv) the quantities of available natural resources.

In correspondence with the underlying long-period competitive position of the economy the capital stock is assumed to be fully adjusted to these data. Hence the 'normal' desired pattern of utilization of plant and equipment would be realized and a uniform rate of return on its supply price obtained. The data or independent variables from which neoclassical theories typically start are the following. They take as given

(a) the set of technical alternatives from which cost-minimizing producers can choose;

(b) the preferences of consumers; and

(c) the initial endowments of the economy and the distribution of property rights among individual agents.

It is easily checked that the given (a) is not very different from the given (i), whereas the given (ii) could be thought of as determined by the given (b). What makes the two theories really different are the data (iii) and (c). However, in the special case in which there is no labour in the economy – and therefore the given (iii) is automatically deleted, because the rate of profit would be endogenously determined and could not be given from outside the system – the given (c) is not very different from the given (iv). It will be shown that it is a characteristic feature of some of the most prominent contributions to the modern literature on endogeneous growth that they eliminate labour from the picture and put in its stead 'human capital' or 'knowledge', that is, something that a twentieth century audience can accept as a producible (and accumulable) factor of production. However, the conditions of production of this surrogate of 'labour' play exactly the same role played in the classical analysis by the assumption of a given real wage rate. This chapter is devoted to a clear statement of this fact.

In this chapter we focus attention on the *analytical structure* of the theory. This does not mean that we are unaware of the fact that there are other elements in the NGT with a decidedly classical flavour. The insistence on increasing returns, for example, bears a close resemblance to Adam Smith's treatment of the division of labour. It was indeed Smith's contention that the accumulation of capital is a prerequisite to the emergence of new and the growth of many of the existing markets which is intimately intertwined with an ever more sophisticated division of labour, and which in turn is seen to be the main source of a continual increase in labour productivity. In Smith's view the division of labour leads to the discovery of new methods and means of production – new machines – and new goods and is generally associated, at least temporarily, with forms of monopolistic competition which allow the successful innovators to reap extra profits for some time (see, for example, Smith, WN, I.x.b.43; see also Young, 1928). Hence in Smith the endogeneity of the rate of growth is not so much the result of the features of some *given* technology, but rather the result of the continuous *revolution* of the technological, organisational and institutional conditions of production, that is a process of the development of the 'productive powers of society'. Whilst we are aware of the similarities between this view and some of the ideas developed in more recent contributions to NGT[1], our main concern in this chapter is not with them but with showing that the set of data from which the majority of NGMs start is that typical of the classical and not that of the neoclassical approach.

Section II shows that Ricardo consistently conceptualised economic growth as endogeneous. In addition, it is shown that the usual Ricardian model can be transformed into one or the other of the conventional NGMs either by eliminating the scarcity of land, or by limiting the effect of the scarcity of land on the rate of profit by means of a backstop technology or by means of increasing returns to scale effects connected with the division of labour. The typology elaborated in this section is then used in Section III in order to analyse and classify some of the more recent NGMs. It is shown that the models under consideration replicate the behaviour of the Ricardesque models investigated in section II. Section IV draws some conclusions.

## II. Endogeneous growth in the 'classical' economists

### 1. Accumulation *vis-à-vis* diminishing returns in agriculture

The problem of economic growth and income distribution was a major concern of Adam Smith and David Ricardo. Ricardo's argument about what he called the 'natural' course of the economy contemplated an economic system in which capital accumulates, the population grows, but there is no technical progress: the latter is set aside. Hence the argument is based on the (implicit) assumption that the set of (constant returns to scale) methods of production from which cost-minimizing producers can choose is given and constant. Assuming the real wage rate of workers to be given and constant, the rate of profit is bound to fall: due to extensive and intensive diminishing returns on land, 'with every increased portion of capital employed on it, there will be a decreased rate of production.' (Ricardo, [1817] 1951, p. 98). Profits are viewed as a residual income based on the surplus product left after the used up means of production and the wage goods in the support of workers have been deducted from the social product (net of rents). The 'decreased rate of production' thus involves a decrease in profitability. On the premise that there are only negligeable savings out of wages and rents, a falling rate of profit involves a falling rate of capital accumulation. Hence, as regards the dynamism of the economy attention should focus on profitability. Assuming that the marginal propensity to accumulate out of profits, s, is given and constant, a 'classical' accumulation function can be formulated

$$g = \begin{cases} s(r - r_{min}) & \text{if } r \geq r_{min} \\ 0 & \text{if } r \leq r_{min} \end{cases}$$

where $r_{min} \geq 0$ is the minimum level of profitability which, if reached, will arrest accumulation (cf. ibid., p. 120). Ricardo's 'natural' course will necessarily end up in a stationary state.[2]

Clearly, in Ricardo the rate of accumulation is endogenously determined. The demand for labour is governed by the pace at which capital accumulates, whereas the long-term supply of labour is regulated by the 'Malthusian Law of Population'.[3]

Assuming for simplicity a given and constant real wage rate, Ricardo's view of the long-run relationship between profitability and accumulation and thus growth can be illustrated in terms of Figure 1, which is a diagram used by Kaldor (1956). The curve CEGH is the marginal productivity of labour-cum-capital; it is decreasing since land is scarce: when labour-cum-capital increases, either less fertile qualities of land must be cultivated or the same qualities of land must be cultivated with processes which require less land per unit of product, but are more costly in terms of labour-cum-capital. Let the real wage rate equal OW. Then, if the amount of labour-cum-capital applied is $L_1$, the area $OCEL_1$ gives the product, $OWDL_1$ gives total capital employed, and BCE total rent. Profit is determined as a residual and corresponds to the rectangular WBED. As a consequence, the *rate* of profit can be determined as the ratio of the areas of two rectangulars which have the same basis and, therefore, it equals the ratio WB/OW. Let us now consider the case in which the amount of labour-cum-capital is larger, that is, $L_2$. Then $OCGL_2$ gives the product, $OWFL_2$ the capital, ACG the rent, and WAGF profits. The rate of profit has fallen to WA/OW. Obviously, if a positive profit rate implies a positive growth rate (i.e., $r_{min} = 0$), the economy will expand until labour-cum-capital has reached the level $\bar{L}$. At that point the profit rate is equal to zero and so is the growth rate. The system has arrived at a stationary state. Growth has come to an end because profitability has.

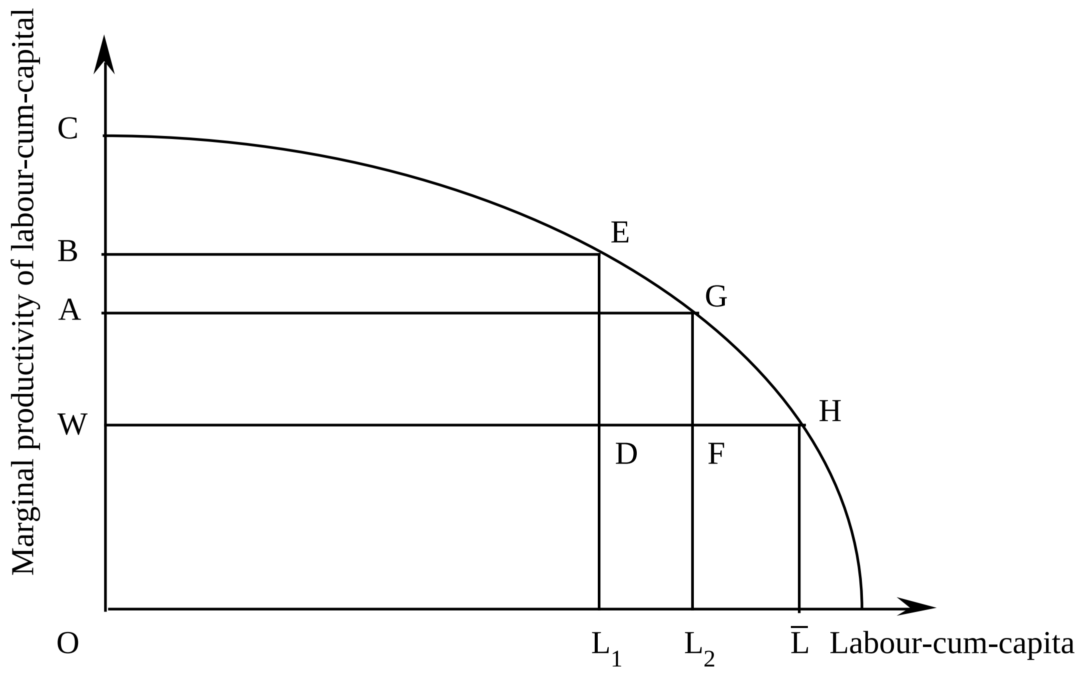


Figure 1: Land as an indispensable resource

The required size of the work force is considered essentially generated by the accumulation process itself. In other words, labour power is treated as a kind of producible commodity. It differs from other commodities in that it is not produced in a capitalistic way by a special industry on a par with other industries, but is the result of the interplay between the generative behaviour of the working population and socio-economic conditions. In the most simple conceptualization possible, labour power is seen to be in elastic supply at a given real wage rate basket. Increasing the number of baskets available in the support of workers involves a proportional increase of the work force. In this view the rate of growth of labour supply adjusts to any given rate of growth of labour demand without necessitating a variation in the real wage rate.[4] Labour can thus put no limit to growth because it is 'generated' within the growth process. The only limit to growth can come from other nonaccumulable factors of production: as Ricardo and others made clear, these factors are natural resources in general and land in particular. In other words, there is only endogenous growth in the classical economists. This growth is bound to lose momentum as the scarcity of natural factors of production makes itself felt in terms of extensive and intensive diminishing returns. (Technical change is of course envisaged to counteract these tendencies.)

## 2. Production with land as a free good

For the sake of the argument let us try to think about Ricardian theory without the problem of land. Setting aside land in Ricardo's doctrine may strike the reader as something similar to Hamlet without the prince. However, the only purpose of this thought experiment is to prepare the ground for a discussion of the NGMs in Section III. If there were no land, or rather: if land of best quality were available in abundance, that is, a free good, then the graph giving the marginal productivity of labour-cum-capital would be a horizontal line and therefore the rate of profit would be constant whatever the amount of labour-cum-capital. This case is illustrated in Figure 2. As a consequence, the growth rate would also be constant: the system could grow forever at a rate that equals the given rate of profit times the propensity to accumulate. As the passage from Ricardo's *Works* placed as the motto of this chapter shows, Ricardo was perfectly aware of this implication.

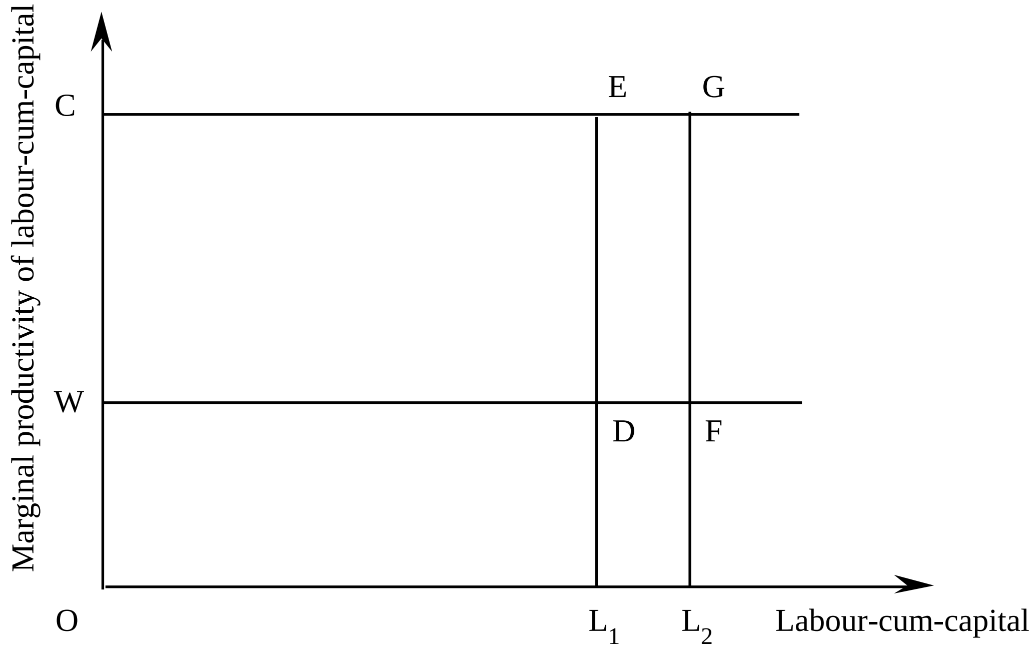


Figure 2: Land as a free good

### 3. Production with a 'backstop technology'

However, to assume that there is no land at all or that it is available in given quality and unlimited quantity is unnecessarily restrictive. With the system growing forever, the point will surely come where land of the best quality gets scarce. This brings us to another constellation in which the rate of profit need not vanish as capital accumulates. The constellation under consideration bears a close resemblance to a case discussed in the economics of 'exhaustible' resources, that is, the case in which there is an ultimate 'backstop technology'. For example, some exhaustible resources are used to produce energy. In addition, there is solar energy which may be considered an undepletable resource. A technology based on the use of solar energy defines the backstop technology mentioned. Let us translate this assumption into the context of a Ricardian model with land.

The case under consideration would correspond to a situation in which 'land', although useful in production, is not indispensable. In other words, there is a technology which allows the production of the commodity without any 'land' input; this is the backstop technology. With continuous substitutability between labour-cum-capital and land, the marginal productivity of labour-cum-capital would be continuously decreasing, but it would be bounded from below. This case is illustrated in Figure 3, with the dashed line giving the lower boundary. In this case the profit rate and thus the growth rate would be falling, but they could never fall below certain levels, which are positive. The system would grow indefinitely at a rate of growth which would asymptotically approach the product of the given saving rate times the value of the (lower) boundary of the profit rate. In Figure 3 the latter is given by WR/OW.

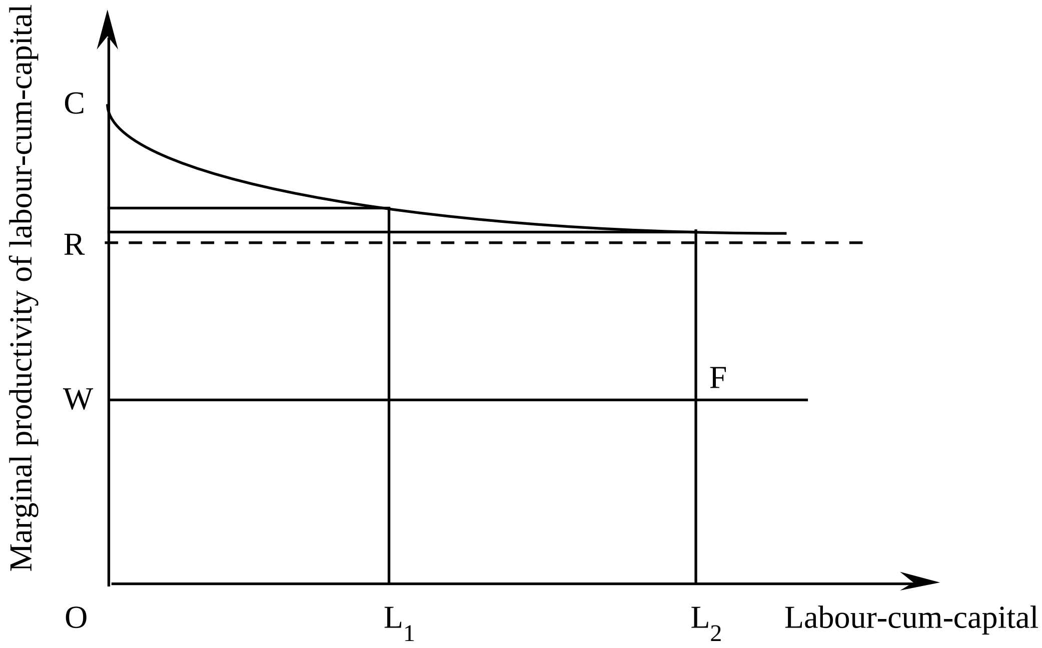


Figure 3: A backstop technology

## 4. Increasing returns to capital-cum-labour

Finally, we may illustrate the case of increasing returns to labour-cum-capital (see Figure 4), as it was discussed, following Adam Smith's analysis of the division of labour, by authors such as Allyn Young (1928) and Nicholas Kaldor (1957). For the sake of simplicity, taking the wage rate as given and constant, the rate of profit and the rate of growth are bound to rise as more labour-cum-capital is employed. (In Figure 4 it is assumed that there is an upper boundary to the rise in output per unit of labour-cum-capital given by OR.) In order to be able to preserve the notion of a uniform rate of profit, it has to be assumed that the increasing returns are *external* to the firm and exclusively connected with the expansion of the market as a whole and the social division of labour. This implies that whereas in the case of decreasing returns due to the scarcity of land (cf. Figures 1 and 3) the product was given by the area under the marginal productivity curve, now the product associated with any given amount of labour-cum-capital is larger than or equal to that amount times the corresponding level of output per unit of labour-cum-capital. It is larger, if there is still scarce land; it is equal to it, if there is not. In any case, the sum of profits and wages equals the product of the given amount of labour-cum-capital times the corresponding level of output per unit of labour-cum-capital.[5] Hence, in the case in which labour-cum-capital is $L_2$, the product is given by the corresponding rectangular. As a consequence, the product is larger than the area under the marginal productivity curve. The cases of decreasing and increasing returns are therefore not symmetrical. It goes without saying that in this case a rising real wage rate need not involve a falling general rate of profit.

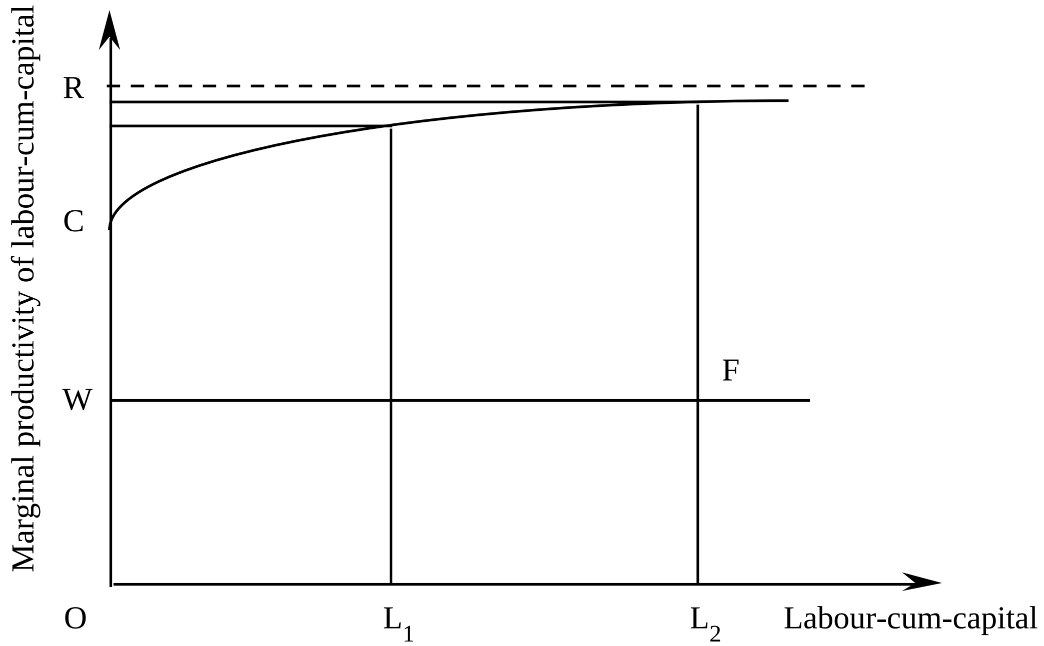


Figure 4: Increasing returns

To conclude, it is to be stressed again that the 'Ricardesque' patterns of endogenous growth illustrated in Figures 1-4 are intimately related to the fact that labour is envisaged as a commodity which is in some sense 'produced' by using corn and nothing else. The real wage rate is considered 'on the same footing as the fuel for the engines or the feed for the cattle'. The straight line WF in the above figures can indeed be interpreted as the 'marginal cost function' related to the 'production' of labour. If the wage rate were to depend on the amount of labour employed, then the marginal cost function would not be a straight line, but substantially the same argument would apply. Put in a nutshell, the 'secret' of the endogeneity of growth in the classical authors consisted of the assumption of a 'technology' producing labour. We will see in Section III that essentially the same secret is at the heart of the NGT.

There is, however, another possibility to interpret the diagrams. In order for this interpretation to hold, we have to remove labour from the scene. If this is done, a picture emerges in which corn is produced by using only corn (including corn used as real wage rate) and, eventually, land. The curve that was previously interpreted as the marginal productivity of labour-cum-capital can now be interpreted as the marginal productivity of corn (as an input); the straight line WF would therefore be located at a distance from the horizontal axis of exactly one unit (WO = 1). All the other elements of the argument developed above would remain exactly the same. We shall see in the following section that this interpretation provides a key to understanding an important aspect of the NGT.

## III. The 'new' growth models

### 1. A classification

As we have seen, the concept of 'endogeneity' employed in the NGMs as specified by Barro and Sala-i-Martin implies that long-run growth is determined 'within the model' rather than by some exogenously growing variables. They add: 'The key property of endogenous-growth models is the absence of diminishing returns to capital' (ibid., p. 39). Therefore, the way or mechanism by means of which diminishing returns to capital are avoided provides a criterion to classify the NGMs (see also Kurz and Salvadori, 1997a). We may distinguish between the following types of models:

(i) 'linear models' or 'AK models' (Rebelo, 1991; King and Rebelo, 1990);

(ii) models in which returns to capital are bounded from below (Jones and Manuelli, 1990);

(iii) the model by Lucas (1988), which focuses attention on the accumulation of human capital; and

(iii) the model by Romer (1986), which emphasises the generation of new knowledge in research and development activities of firms.

We shall deal with these different models in turn.

## 2. Linear or 'AK' models

First, there are models which set aside all *nonaccumulable* factors of production such as labour and land and assume that all inputs in production are accumulable, that is, 'capital' of some kind. The simplest version of this class of models is the so-called 'AK model', which assumes that there is a linear relationship between total output, Y, and a single factor capital, K, both consisting of the *same* commodity:

$$Y = AK, \tag{1}$$

where 1/A is the amount of that commodity required to produce one unit of itself. Because of the linear form of the aggregate production function, these models are also known as 'linear models'. The rate of return on capital r is given by

$$r + \delta = \frac{Y}{K} = A, \tag{2}$$

where $\delta$ is the exogenously given rate of depreciation. The saving-investment mechanism jointly with the assumption of a uniform rate of growth, that is, a steady-state equilibrium, then determine a relationship between the growth rate, g, and the rate of profit, r. Rebelo (1991, pp. 504 and 506) obtains either

$$g = \frac{A - \delta - \rho}{\sigma} = \frac{r - \rho}{\sigma}, \qquad (3)$$

or

$$g = (A - \delta)s = sr. \qquad (4)$$

Equation (3) is obtained when savings are determined on the assumption that there is an immortal representative agent maximizing the following intertemporal utility function

$$\int_0^\infty e^{-\rho t} \frac{1}{1 - \sigma} [c(t)^{1-\sigma} - 1]\, dt, \qquad (5)$$

subject to constraint (1), where $\rho$ is the discount rate, or rate of time preference, and $1/\sigma$ is the elasticity of substitution between present and future consumption ($1 \neq \sigma > 0$), and where $Y = c(t) + \dot{K}$, where $\dot{K}$ is the derivative of K with respect to time, i.e., investment. Equation (4) is obtained when the average propensity to save s is given. Hence, in this model the rate of profit is determined by technology alone and the saving-investment mechanism determines the growth rate.

This model is immediately recognized as the model dealt with at the end of Subsection II.4, in which labour was set aside, on the assumption that the technology to produce corn is that illustrated in Figure 2. Even the saving-investment mechanism is essentially the same: in the case of equation (3) $\sigma = 1/s$ and $\rho = r_{min}$ (provided that $r > r_{min}$); in the case of equation (4) $r_{min} = 0$. Hence, the version of the 'new' growth theory under consideration is but the most elementary of all classical models. No classical economist can be accused of having taken that model too seriously.

A slightly different avenue was followed by King and Rebelo (1990). Instead of one kind of 'capital' they assumed that there are two kinds, real capital and human capital, both of which are accumulable. There are two lines of production, one for the social product and the real capital, which consist of quantities of the same commodity, and one for human capital. The production functions relating to the two kinds of capital are assumed to be homogeneous of degree one and strictly concave. There are no diminishing returns to (composite) capital for the reason that there is no nonaccumulable factor such as simple or unskilled labour that enters into the production of the accumulable factors, investment goods and human capital.[6] The production functions relating to the two kinds of capital are given by

$$H = H(H_H, K_H) \tag{6.1}$$

and

$$K = K(H_K, K_K). \tag{6.2}$$

As in Rebelo's model the rate of profit is uniquely determined by the technology (and the maximisation of profits which implies that only one technique can be used in the long run); the growth rate of the system is then endogenously determined by the saving-investment equation. Maximisation of profits implies that

$$\frac{\partial H}{\partial H_H} = r \tag{7.1}$$

$$\frac{\partial H}{\partial K_H} = \frac{r}{p} \tag{7.2}$$

$$\frac{\partial K}{\partial H_k} = rp \tag{7.3}$$

$$\frac{\partial K}{\partial K_K} = r, \tag{7.4}$$

where r is the rate of profit and p is the price of human capital in terms of the commodity which is consumed or accumulated as physical capital ($\delta$ has been set equal to 0 in order to simplify the notation). Since functions (6) are homogeneous of degree one, their first derivatives are homogeneous of degree zero, and hence the four equations (7) are enough to determine the four unknowns r, p, $H_H/K_H$, $H_K/K_K$.[7] This is nothing else than the Nonsubstitution Theorem[8], which, as is well known, implies that only one technique can be used in the long run. The growth rate of the system is then endogenously determined by the saving-investment equation. The larger the propensities to accumulate human and physical capital, the larger is the growth rate.

Comparing the latter model with the classical theory we can draw the following conclusion: the role played by 'labour' in the classical authors is assumed by 'human capital' in King and Rebelo (1990). Both factors of production are taken to be producible; with constant returns to scale, as in King and Rebelo (1990) and in the case depicted in Figure 2, the rate of profit and, therefore, the rate of growth are determined and constant over time. The linear NGMs thus simply replicate in elementary terms the logic of the classical approach to the theory of distribution and growth.

## 3. Returns to capital bounded from below

Next there are models which preserve the dualism of accumulable and nonaccumulable factors but restrict the impact of an accumulation of the former on their returns by a modification of the aggregate production function. Jones and Manuelli (1990), for example, allow for both labour and capital and even assume a convex technology, as the Solow model (cf. Solow, 1956). However, a convex technology requires only that the marginal product of capital is a decreasing function of its stock, not that it vanishes as the amount of capital per worker tends to infinity. Jones and Manuelli assume that

$$h(k) \geq bk, \qquad \text{each } k \geq 0,$$

where h(k) is the per capita production function and b is a positive constant. The special case contemplated by them is

$$h(k) = f(k) + bk, \tag{8}$$

where f(k) is the conventional Solovian production function. As capital accumulates and the capital-labour ratio rises, the marginal product of capital will fall, approaching asymptotically b – its lower boundary. With a given propensity to save s and assuming capital to be everlasting, the steady-state growth rate g is endogenously determined: g = sb. Assuming on the contrary intertemporal utility maximization, the rate of growth is positive provided the technical parameter b is larger than the rate of time preference $\rho$. In the case in which it is, the steady-state rate of growth is given by (3) with r = b.

It is not difficult to recognize that the difference between the model of Jones and Manuelli (1990) and that of Rebelo (1991) is the same as the one existing between the cases illustrated by Figures 3 and 2 above.

Finally, there is a large class of models which contemplate various factors counteracting any diminishing tendency of returns to capital. The models can be grouped in two. In both kinds of models *positive external effects* play an important part: they offset any fall in the marginal product of capital.

## 4. Human capital formation and its externalities

Models of the first group attempt to formalize the role of human capital formation in the process of growth. Elaborating on some ideas of Uzawa (1965), Lucas (1988) assumed that agents have a choice between two ways of spending their (non-leisure) time: to contribute to current production or to accumulate human capital. It is essentially the allocation of time between the two alternatives contemplated that decides the growth rate of the system. For example, a decrease in the time spent producing goods involves a reduction in current output; at the same time it speeds up the formation of human capital and thereby increases output growth. With the accumulation of human capital there is said to be associated an externality: the more human capital society as a whole has accumulated, the more productive each single member will be. This is reflected in the following macroeconomic production function

$$Y = AK^{\beta}(uhN)^{1-\beta}h^{*\gamma}, \qquad (9)$$

where the labour input consists of the number of workers, N, times the fraction of time spent working, u, times h which gives the labour input in efficiency units. Finally, there is the term h*. This is designed to represent the externality. The single agent takes h* as a parameter in his or her optimizing by choice of consumption c and u. However, for society as a whole the accumulation of human capital increases output both directly and indirectly, that is, through the externality. Here we are confronted with a variant of a *public good* problem, which may be expressed as follows. The individual optimizing agent faces constant returns to scale in production: the sum of the partial elasticities of production of the factors he or she can control, that is, his or her physical and human capital, is unity. Yet for society as a whole the partial elasticity of production of human capital is not $1 - \beta$, but $1 - \beta + \gamma$.

Lucas's conceptualisation of the process by means of which human capital is built up is the following:

$$\dot{h} = \upsilon h(1 - u), \tag{10}$$

where $\upsilon$ is a positive constant. (Note that equation (10) can be interpreted as a 'production function' of human capital by means of human capital: the average product is constant and equals $\upsilon$.)

Interestingly, it can be shown that if there is *not* the above mentioned externality, i.e., if $\gamma$ in equation (9) equals zero, and therefore returns to scale are constant and, as a consequence, the Nonsubstitution Theorem holds, endogenous growth in Lucas's model is obtained in essentially the same way as in the models by Rebelo (1991) and King and Rebelo (1990): the rate of profit is determined by technology and profit maximisation alone; and for the predetermined level of the rate of profit the saving-investment mechanism determines the rate of growth. Hence, as Lucas himself pointed out, the endogenous growth is positive *independently* of the fact that there is the above mentioned externality, that is, independently of the fact that $\gamma$ is positive.

With the "production functions" (9) and (10), and $\gamma = 0$, profits are maximised when

$$w_e = p\upsilon \tag{11.1}$$

$$r = \beta A\left(\frac{K}{uhN}\right)^{\beta-1} \tag{11.2}$$

$$w_e = (1 - \beta)A\left(\frac{K}{uhN}\right)^{\beta}, \qquad (11.3)$$

where $w_e$ is the wage per efficiency unit of labour (if $w_h$ is the hourly wage of a worker of skill h, then $w_h = w_e h$), p is the price of human capital in terms of the single commodity that is consumed or accumulated as physical capital, and r is the rate of profit. In conditions of free competition the rate of profit tends to be uniform across the two sectors. This implies that the existing human capital times the rate of profit equals the income obtained from that human capital, that is,

$$rNhp = w_e uNh + \dot{N}hp + N\dot{h}p + Nh\dot{p} \qquad (12)$$

Since the Nonsubstitution Theorem holds, p and $w_e$ are uniquely determined in the long run and, therefore, in steady states $\dot{p} = 0$. Then, from equations (10), (11.1) and (12) we obtain

$$r = \upsilon + \lambda,$$

where $\lambda$ is the *exogenous* rate of growth of population. There is only one meaning that can be given to the dependence of r on $\lambda$: it is a consequence of the remarkable fact that in Lucas's model the growth of 'population' means simply that the immortal consumer grows 'bigger' at rate $\lambda$. (Otherwise one would have to assume the existence of another type of externality: costless cultural transmission, that is, to new generations the existing knowledge is a free good.) Thus, as in Rebelo's model, the rate of profit is determined by technology (and profit maximisation) alone. Equations (11.2) and (11.3) determine the technique utilised in the commodity producing sector and the wage rate:

$$\frac{K}{uhN} = \left(\frac{\upsilon + \lambda}{\beta A}\right)^{(\beta-1)^{-1}}$$

$$w_e = (1 - \beta)A\left(\frac{\upsilon + \lambda}{\beta A}\right)^{\beta(\beta-1)^{-1}}.$$

Hence, if u is constant over time, and K, h, and N grow at rates that are also constant over time, that is, the economy is in a steady state, then

$$\frac{\dot{K}}{K} = \frac{\dot{h}}{h} + \frac{\dot{N}}{N}.$$

Finally, as in the models of Rebelo (1991) and King and Rebelo (1990), the behaviour of consumers (and investors) reflected in the saving-investment equation determines a relationship between the rate of profit and the rate of growth, and since the profit rate is determined by technology (and the choice of technique), the growth rate is *endogenously* fixed. With Lucas's assumptions about saving

$$\frac{\dot{h}}{h} = \frac{r - \rho}{\sigma}, \qquad (13)$$

that is

$$\frac{\dot{h}}{h} = \frac{\upsilon + \lambda - \rho}{\sigma},$$

which implies that

$$u = \frac{\upsilon(\sigma - 1) + \rho - \lambda}{\upsilon\sigma},$$

and since $0 \leq u \leq 1$

$$0 \leq \frac{\upsilon + \lambda - \rho}{\sigma} \leq \upsilon.$$

Let us now assume a positive $\gamma$ (but lower than $(1 - \beta)\sigma$). In this case returns to scale are not constant. Hence, the Nonsubstitution Theorem does not apply, and this is the reason why neither the profit maximising technique, nor $w_e$, nor p are determined by technology and profit maximisation alone. As a consequence also r is not so determined. The simple 'recursive' structure of the model is thereby lost. Nevertheless, technology and profit maximisation still determine, in steady states, a *relationship* between the rate of profit and the rate of growth. This relationship together with the relationship between the same rates obtained from the saving-investment equation determine both variables. Thus, although the analysis is more complex, essentially the same mechanism applies.

In fact, if $\gamma > 0$, equations (11) become:

$$w_e = p\upsilon \qquad (14.1)$$

$$r = \beta A h^{\gamma} \left(\frac{K}{uhN}\right)^{\beta-1} \qquad (14.2)$$

$$w_e = (1 - \beta) A h^{\gamma} \left(\frac{K}{uhN}\right)^{\beta}. \qquad (14.3)$$

From equations (14.1) and (14.3) we obtain

$$\frac{\dot{w}_e}{w_e} = \frac{\dot{p}}{p}$$

$$\frac{\dot{w}_e}{w_e} = (\gamma - \beta)\frac{\dot{h}}{h} + \beta\left(\frac{\dot{K}}{K} - \lambda\right).$$

From the production function (9) we obtain that in steady states

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{1 - \beta + \gamma}{1 - \beta}\frac{\dot{h}}{h} + \lambda.$$

Hence

$$\frac{\dot{p}}{p} = \frac{\dot{w}_e}{w_e} = \frac{\gamma}{1 - \beta}\frac{\dot{h}}{h},$$

which, substituted in equation (12) and taking account of equations (10) and (14.1), gives

$$r = \upsilon + \lambda + \frac{\gamma}{1 - \beta}\frac{\dot{h}}{h},$$

which jointly with equation (13) determines both the growth rate and the rate of profit:

$$r = \frac{(1 - \beta)\sigma(\nu + \lambda) - \gamma\rho}{(1 - \beta)\sigma - \gamma}$$

$$\frac{\dot{h}}{h} = \frac{(1 - \beta)(\nu + \lambda - \rho)}{(1 - \beta)\sigma - \gamma} .$$

Thus, although the analysis is more complex, essentially the same mechanism applies as in the models dealt with in Subsection III.2. Once again the concept of 'human capital' has assumed a role equivalent to the role of the concept of 'labour' in classical economics. However, while most contemporary economists would presumably be hostile to the idea that 'labour' could be treated as a produced factor of production, they appear to have had no difficulty in accepting the idea that there is a technology producing 'human capital'.[9]

We want to stress that the results obtained in this section are not different from those obtained by Lucas (1988) by using his procedure of maximizing the funtional (5) subject to the the constraints (9) and (10) and then assuming that the available amounts of human capital and physical capital are those which allow the steady state. However, we arrived at the results much more easily since our analysis was a long period one from the beginning and this gave us directly equations (11) and (12) in the case of constant returns to scale (and (14) and (12) in the case of increasing returns to scale). The results are obtained when these equations are put together with equation (13) – an equation that Lucas obtained by the assumption of an everlasting consumer, but that can be obtained also otherwise. This should clarify the *detour* aspect of the intertemporal analysis with respect to the long-period one, when we are interested in the 'balanced path', considered 'as a good approximation to any actual path "most" of the time' (ibid., p. 11).

## 5. Endogenous technical change

Models of the second group attempt to portray technological change as generated endogenously. The proximate starting point of this kind of models was Arrow's paper on 'learning by doing' (Arrow, 1962). In Romer (1986) attention focuses on the role of a single state variable called 'knowledge' or 'information'. It is assumed that the information contained in inventions and discoveries has the property of being available to anybody to make use of it at the same time. In other words, information is considered essentially a nonrival good. However, it need not be totally nonexcludable, that is, it can be monopolized at least for some time. It is around the two different aspects of publicness (nonrivalry and nonexcludability) that the argument revolves. Discoveries are made in research and development departments of firms. This requires that resources be withheld from producing current output. The basic idea of Romer's model is 'that there is a trade-off between consumption today and knowledge that can be used to produce more consumption tomorrow' (ibid., p. 1015). He formalizes this idea in terms of a 'research technology' that produces 'knowledge' from foregone consumption. Knowledge is assumed to be cardinally measurable and not to depreciate: it is like perennial capital.

Romer stipulates a research technology that is concave and homogeneous of degree one,

$$\dot{k}_i = G(I_i, k_i), \qquad (15)$$

where $I_i$ is an amount of foregone consumption in research by firm i and $k_i$ is the firm's current stock of knowledge. (Note that equation (15) can be interpreted as a production function describing the production of 'knowledge' by means of 'knowledge' and the foregone consumption good.) The production function of the consumption good relative to firm i is

$$Y_i = F(k_i, K, \mathbf{x}_i), \qquad (16)$$

where K is the accumulated stock of knowledge in the economy as a whole and $\mathbf{x}_i$ is a vector of inputs different from knowledge. Romer assumes that 'factors other than knowledge are in fixed supply' (ibid., p. 1019). This implies that 'knowledge' is the only capital good utilised in the production of the consumption good. (The foregone consumption good is a capital good used in the production of knowledge.) Spillovers from private research and development activities increase the public stock of knowledge K. It is assumed that the function is homogeneous of degree one in $k_i$ and $\mathbf{x}_i$ and homogeneous of a degree greater than one in $k_i$ and K.

Assuming, contrary to Romer, that the above production function (16) is homogeneous of degree one in $k_i$ and K involves a constant marginal product of capital: the diminishing returns to $k_i$ are exactly offset by the external improvements in technology associated with capital accumulation. In this case it can be shown that as in the models previously dealt with, the rate of profit is determined by technology and profit maximization alone, provided, as is assumed by Romer, that the ratio $K/k_i$ equals the (given) number of firms.

In fact, profit maximisation requires that

$$p \frac{\partial G}{\partial I_i} = r \qquad (17.1)$$

$$\frac{\partial G}{\partial k_i} = r \qquad (17.2)$$

$$\frac{\partial F}{\partial k_i} = rp \qquad (17.3)$$

$$\frac{\partial F}{\partial x_{ij}} = w_j, \qquad (18)$$

where p is the price of 'knowledge' in terms of the consumption good and $w_j$ is the rental of the j-th fixed factor. The derivative of $F(k_i, K, \mathbf{x}_i)$ with respect to $k_i$ is homogeneous of degree zero in $k_i$ and K. Then it depends only on the given vector $\mathbf{x}_i$ and the ratio $K/k_i$, which, since all firms are taken to be equal to one another, coincides with the (given) number of firms S. That is, since $\mathbf{x}_i$ is a given vector and since function (15) is homogeneous of degree one, the three equations (17) involve only three unknowns: r, p, $I_i/k_i$. As in the models previously dealt with, the rate of profit is determined by technology and profit maximisation alone, so that the saving-investment relation can determine the growth rate *endogenously*. (Equations (18) just determine the rentals of the fixed factors.)

Once again endogenous growth does not depend on an assumption about increasing returns with regard to accumulable factors. Growth would not be 'more endogenous' if increasing returns were to be assumed. Such an assumption renders the analysis a good deal more complicated. In particular, a steady-state equilibrium does not exist unless the marginal product of capital is taken to be bounded from above. This is done by Romer in terms of an *ad hoc* assumption regarding equation (15) (ibid., p. 1019). This assumption is not different from the one used in drawing Figure 4 above, where the marginal product of labour-cum-capital is shown to be increasing with the scale of production, but is bounded from above.

## IV. Concluding remarks

The chapter has shown how in some of the most well-known NGMs endogenous growth is generated. Notwithstanding their many differences, it is a striking common feature of these models that the rate of profit is determined by technology alone, or, if there is a choice of technique, by the profit maximising behaviour of producers. With the rate of profit determined in this way, the task of the saving-investment mechanism is restricted to the determination of the steady-state growth rate. With a given saving rate, the growth rate is simply the profit rate times the saving rate. With intertemporal utility maximisation things are slightly more complicated and the saving rate is endogenously determined. It has also been shown that increasing returns are not an indispensable ingredient in order to obtain endogenous growth. The profit rate is determined by technology because it is assumed that there is a technology producing 'labour'. In order to render this fact acceptable to a twentieth century audience, the factor has been given new names and enters the stage either as 'human capital' or 'knowledge'. Exactly as in the Ricardian analysis, in this way the profit rate is determined. The readers of *Production of Commodities by Means of Commodities* by Piero Sraffa (1960) will immediately recall that when at the beginning of chapter II (§§ 4-5) wages are regarded as entering the system 'on the same footing as the fuel for the engines or the feed for the cattle', the profit rate and the prices are determined by technology alone. On the contrary, when workers get a part of the surplus, the quantity of labour employed in each industry has to be represented explicitly, and the profit rate and the prices can be determined only if an extra equation determining income distribution is introduced into the anlysis. The additional equation generally used by advocates of neoclassical analysis is the equality between demand and supply of 'capital', which requires the homogeneity of this factor.[10] But no extra equation is required in the NGT since, as in Ricardo and in §§ 4-5 of Sraffa's book, there is a technology producing 'labour'.

Finally, it should be noted that the NGT has revived *long-period* analysis, centred around the concept of a uniform rate of profit. However, the kind of long-period argument put forward in the NGT falls way behind the present state of the art in this field of research. In particular, it appears to us to be anachronistic to attempt to develop a theory of growth that focuses on product innovations, new 'industrial designs' etc. in terms of a model which preserves several of the disquieting features of the neoclassical growth theory of the 1950s and 1960s, including the setting aside of the diversity of behaviour and the heterogeneity of goods and particularly of capital goods. These latter assumptions the NGT shares with Knight's famous Crusonia plant, in particular, a homogeneous capital jelly (cf. Kurz and Salvadori, 1997a). There is no need and indeed no justification to continue to dwell on such fairy tales. First, because the structure of the theory does not require such an assumption since distribution is *not* determined by the equality of the demand and supply of 'capital'. Second, because modern long-period theory of 'classical' derivation might offer an alternative that allows a better understanding of the phenomena under consideration.

We hope to have shown that many of the interesting aspects of the NGMs are related to the classical perspective their authors (unwittingly) take on the problem of growth, whereas some of their shortcomings derive from the lack of solutions to the problems of the neoclassical theory of growth which were put into sharp relief during the 1960s.

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1 See, for example, Yang and Borland (1991), Becker and Murphy (1992), Rodriguez-Clare (1996); see also the so-called neo-Schumpeterian models, e.g., Aghion and Howitt (1992).

2 This path must, of course, not be identified with the *actual* path the economy is taking because technical progress will repeatedly offset the impact of the 'niggardliness of nature' on the rate of profit.

3 Real wages may rise, that is, the 'market price of labour' may rise above the 'natural' wage rate. This is the case in a situation in which capital accumulates rapidly, leading to an excess demand for labour. As Ricardo put it, 'notwithstanding the tendency of wages to conform to their natural rate, their market rate may, in an improving society, for an indefinite period, be constantly above it' (ibid., pp. 94-5). If such a constellation prevails for some time it is even possible that 'custom renders absolute necessaries' what in the past had been comforts or luxuries. Hence, the natural wage is driven upward by persistently high levels of the actual wage rate. Accordingly, the concept of 'natural wage' in Ricardo is a flexible one and must not be mistaken for a physiological minimum of subsistence. For Smith's view on wages and the growth of the work force, see Kurz and Salvadori (1995, ch. 15).

4 In the more sophisticated conceptualizations underlying the arguments of Smith and Ricardo, higher rates of growth of labour supply presuppose higher levels of the real wage rate. But the basic logic remains the same: in normal conditions the pace at which capital accumulates regulates the pace at which labour grows.

5 Let $x = f(L, L^*)$ be the product of the last unit of labour-cum-capital when L represents the amount of labour-cum-capital employed and the division of labour is artificially kept fixed at the level appropriate when the amount of labour-cum-capital employed is L*. Obviously, $f(L, L^*)$ as a function of L alone is either decreasing as in Figures 1 and 3 (if land is scarce) or constant as in Figure 2 (if land is not scarce). The product at L* equals $\int_0^{L^*} f(L, L^*)dL$, that is, the area under the curve $f(L, L^*)$ in the range [0, L*]. If $\frac{\partial f}{\partial L^*} > -\frac{\partial f}{\partial L}$ for $L = L^*$, then the curve

$$x = f(L, L),$$

which is the curve depicted in Figure 4, is increasing, but the product is, as stated in the text, larger than or equal to the sum of profits and wages, which equals the product of the given amount of labour-cum-capital times the corresponding level of output per unit of labour-cum-capital.

6 The assumption that the formation of human capital does not involve any unskilled labour as an input is not convincing: the whole point of education processes is that a person's capacity to perform unskilled labour is gradually transformed into his or her capacity to perform skilled labour. Adam Smith, for example, was perfectly aware of this. For an analytical treatment of the problem of human capital, taking Smith's discussion as a starting point, see Kurz and Salvadori (1995, ch. 11).

7 It is easily checked that if the production functions (6) are 'well-behaved', then there is one and only one solution to system (7).

8 We need a special case of the Nonsubstitution Theorem, because no primary factor (or a primary factor with a zero remuneration) is assumed; see Kurz and Salvadori (1994), reproduced here as Chapter 4.

9 It is possible to show that the Lucas model can easily be generalised to take into account non produced means of production. If land, Q, is introduced so that the production function (9) becomes

$$Y = AK^{\beta}(uhN)^{\alpha}Q^{1-\alpha-\beta}h^{*\gamma},$$

by following the above procedure we obtain

$$r = \upsilon + \frac{\alpha}{1-\beta}\lambda + \frac{\alpha+\beta+\gamma-1}{1-\beta}\frac{\dot{h}}{h}.$$

Note that if $\alpha + \beta + \gamma = 1$, that is, if returns to scale with respect to accumulable factors are constant, then the rate of profit is determined by technology and profit maximisation alone; otherwise technology and profit maximisation determine a linear relationship between the rate of profit and the rate of growth.

10 This is the famous critique of that theory put forward in the 1960s; for a review of that critique, see Harcourt (1972) and Kurz and Salvadori (1995, ch. 14).