

Removing an 'Insuperable Obstacle' in the Way of an Objectivist Analysis: Sraffa's Attempts at Fixed Capital*

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We must prove that the only quantities whose knowledge is required to determine prices are the Capital Inputs; while the Capital Stock is not required, and anyhow is only a derived quantity, derived from the inputs. (D3/12/27: 46 (5))

1. Introduction

Sraffa's (re-)constructive and critical work which culminated in the publication of *Production of Commodities by Means of Commodities* (1960) was carried out for the main part in three

* The present paper derives from Kurz (2003) and Kurz and Salvadori (2004a). We should like to thank Pierangelo Garegnani, literary executor of Sraffa's papers and correspondence, for granting us permission to quote from them. References to the papers which are kept at Trinity College Library, Cambridge, follow the catalogue prepared by Jonathan Smith, archivist. Unless otherwise stated, all emphases are in the original, where words or passages Sraffa underlined are italicised by us. Sraffa frequently abbreviated 'and' by '+'; we shall use the word instead of the symbol. Since in his texts Sraffa used both round and square brackets, all additions by us will be in wavy brackets, { }. (In formulas the latter kind of brackets are Sraffa's.) We should like to thank Jonathan Smith and the staff of Trinity College Library for continuous assistance while working on the Sraffa papers. In addition we should like to thank Pierangelo Garegnani, Christian Gehrke, Harvey Gram, Geoff Harcourt, Marco Lippi, Cristina Marcuzzo, Nerio Naldi, Bertram Schefold, Ian Steedman and two referees of *EJHET* for valuable comments and suggestions.

periods of time which were widely separated from each other. He began his work in summer 1927 while preparing his 'Lectures on Advanced Value Theory' which he was to give from 1928 until 1930 in Cambridge. He had to interrupt his respective studies in 1931, when it turned out that the task of editing *The Works and Correspondence of David Ricardo* to which he had been appointed by the Royal Economic Society at the beginning of 1930 absorbed all his energy and did not allow for the parallel pursuance of his own project. Shortly after the period of internment on the Isle of Man (June-October 1940) Sraffa took up the argument where he had left it at the beginning of the 1930s and worked with great excitement and untiring effort on a reformulation of the classical theory of value and distribution. The early 1940s saw him compose a huge number of notes and manuscripts analysing the various aspects of the project under consideration and relate his studies to the works of both major classical and marginalist authors. Alas, he had to interrupt his work again in the second half of the 1940s. The unexpected find in July 1943 of the whole series of Ricardo's letters to James Mill as well as a number of additional writings of Ricardo which had also been in Mill's possession necessitated major editorial changes and turned out to be incompatible with the work on his book. It was only after the manuscript of Volume X, 'Biographical Miscellany', had been sent to the printer in spring 1954 that Sraffa could return to his old notes and finally put together his book from 1955 to January 1958.

Right from the beginning of his constructive work Sraffa struggled with such intricate problems as fixed capital (including capital utilization), natural resources, in particular land, and joint production – problems that were to figure prominently in his book.¹ Especially the problem of fixed capital turned out to be a veritable touchstone to his objectivist approach to the theory of value and distribution: Whereas in the case of circulating capital goods, such as raw materials, the process of value transfer to the product and the physical 'destruction' of the input are one and the same thing, in the case of fixed capital goods, such as machines, this is typically not so.² This endangered Sraffa's attempt at explaining relative prices and one

¹ These were not, of course, the only problems attracting Sraffa's attention: the scope of his philosophical interests, intellectual fascinations and social passion was much wider; see, for example, the brief overview in Kurz (1998).

² Sraffa's concept of 'objectivism' in the theory of value and distribution changed over time. For the purpose of the present essay the concept used is well expressed by the motto at the beginning of the paper and document D3/12/27: 11-12 quoted in Section 6 below.

of the distributive variables (the rate of profits or wages) strictly 'in material terms (i.e. quantities of labour, of commodities and periods of time)' (D3/12/27: 11). The paper analyses Sraffa's consecutive attempts to come to grips with this essential problem, on which his research programme stood or fell. Given the amount of material available in Sraffa's papers, the treatment of the subject in a relatively short paper like this cannot be exhaustive. Attention will focus on what might be considered some of the more important steps on Sraffa's way to the eleven pages of Chapter X, 'Fixed Capital'.

The composition of the paper is the following. Section 2 deals with Sraffa's early distinctions between fixed and circulating capital and their relationship with those advocated by Adam Smith and David Ricardo, respectively. Section 3 turns to Sraffa's attempt in the late 1920s to reduce fixed capital to circulating capital on the one hand and land on the other. In this way he tried to carry over the purely objectivist approach to the theory of value and distribution he had successfully developed for the case of circulating capital in 1927-1928 to the case of fixed capital, using the analytical tools already at his disposal. However, he soon saw that this reduction exhibited serious shortcomings and that some other method had to be found. Section 4 deals with Sraffa's critical attitude in the first period of his constructive work towards the joint products-approach to fixed capital which he encountered in the writings of, among others, Marshall, Marx, and James Mill and was later to adopt himself. Section 5 summarises Sraffa's approach to fixed capital during the first period (focusing on the time from late 1928 to 1931) and then after the resumption of his work on the problem at the beginning of the second period (summer and autumn of 1942). The issues dealt with include: Sraffa's abandonment of the idea that no interest must be allotted to fixed capital; his hypothesis put forward as early as the beginning of 1931 that with a circular flow of production the ratio of the value of the net product to that of social capital (comprising circulating and fixed capital) can be assumed to be independent of the rate of interest; his view, which he was soon to abandon, that production in 'Industry', which can be organised continuously, has to be treated differently from production in 'Agriculture'; the collaboration with his 'mathematical friend' Abram S. Besicovitch who assisted him with ascertaining the mathematical properties of the fixed capital models he elaborated;³ and Sraffa's disenchantment with his efforts up till then because they appeared to undermine his objectivist approach. Section 6 reports on Sraffa's break-through on the problem of fixed capital in the winter of 1942-1943. The starting point of this break-through was his insight in

³ For a detailed account of the collaboration of the two, see Kurz and Salvadori (2004a).

October 1942 that in order to preserve his objectivist approach fixed capital had to be reduced to circulating capital: With the help of Besicovitch he was then able to show that what hitherto had seemed to be an 'insuperable obstacle', using Wicksell's term (see, for example, D3/12/27: 47 and D3/12/36: 6), could be overcome in terms of the joint products-method. In particular, the production and circulation of commodities can be represented exclusively 'in material terms (i.e. quantities of labour, of commodities and periods of time) *independent* of the distribution of the product, i.e. of the rate of profit.' Section 7 concludes.

2. Distinguishing between fixed and circulating capital: the classical heritage

According to Adam Smith (WN, II.i.4-5) *circulating capital* is capital which 'only by means of circulation, or successive exchanges' yields a profit; as wheat, cattle or merchandise. *Fixed capital* instead is capital which yields a profit by remaining in the hands of one owner; as improvements of land, machines, instruments of trade and seed. Sraffa appears to have come across Smith's distinction first in the secondary literature and only after he had developed a similar concept himself.⁴ In the winter of 1927-28 he pointed out with regard to Smith's definition: 'This is exactly my definition' (D3/12/10: 34). A large part of Sraffa's early notes on fixed capital concern exclusively the case of seed, where by seed he generally meant the input of a commodity into its own production, viz. the use of seed corn to produce corn, but also, for example, the input of iron in the production of iron.⁵ Sraffa endorsed

⁴ He referred to Gonner's introduction to the latter's edition of Ricardo's *Principles* (Ricardo 1891) and to Whitaker (1904). However, a most attentive student of Marshall's *Principles*, Sraffa came across the definitions of major classical economists reported in Marshall ([1870] 1977: 63-4; see also Sraffa's copy of Marshall (1922: 75)).

⁵ Sraffa encountered this generalised concept of seed in the French edition of *Theorien über den Mehrwert* (whose publication in altogether eight volumes had been completed by 1925); see Marx (1924, vol. I: 313; see also 250). Interestingly, the idea of seed as fixed capital was also advocated, as Sraffa noted acutely, by Pareto in the *Manuale*; see

Smith's view that 'No fixed capital can yield any revenue but by means of a circulating capital' (WN, II.i.25), that is raw materials and the maintenance of the workmen.

There is no space to provide a detailed account of why Sraffa at first, in 1927, accepted the 'seed' concept of fixed capital, but already a few months later, in 1928, rejected it (see, therefore, Kurz 2003). We also cannot enter into a discussion of the development of what Sraffa called his 'first' and 'second' equations, respectively, that is, those relating to a system without surplus, or, as Sraffa was to call it in chapter I of his 1960 book: 'Production for Subsistence', and to a system with a surplus and wages at the subsistence level as analysed in sections 4-6 of chapter II of the book. Here it must suffice to draw the reader's attention to the fact that initially Sraffa saw reason not to reckon interest on the input of a commodity in its own production, i.e. seed. He saw this assumption justified by the following consideration. The input of a commodity into its *own* production does not 'change hand', i.e. circulate, and therefore should be treated as fixed capital. To this he added the following considerations which implied that there should be no surplus distributed with respect to seed and fixed capital. First, 'the starting point should be that seed does not enter into exchange, and its value is never determined: and since it is only through exchange{,} not through production{,} that individuals get hold of the surplus, no surplus is to be got from seed' (D3/12/6: 18). Secondly, this view conformed with the views of Smith, Ricardo and Marx that machinery was not producing any value but only transmitting its value over time to the flow of output generated with its assistance (see D3/12/10: 66 and D3/12/11: 34).⁶

Pareto (1906: 294-5) and Sraffa's underlined note on the last page of his copy: 'Semente = capitale fisso' with page reference.

⁶ However, in a document of November 1927 Sraffa criticized Ricardo and Marx for considering profits as 'proportional to labour (i.e. food) instead of saying that they are proportional to circulating capital (i.e. food *plus* all materials used up in production). Marx's "variable capital" is *only* wages: it ought to be "circulating capital" in current sense of the word.' (D3/12/11: 64) In a related piece he added: 'It would be a fatal error to give any metaphysical or causal meaning to the fact that circulating capital "produces" and fixed capital and labour "do not produce" a surplus, and are therefore not "entitled" to share in its distribution. – This impression depends only from {sic} the terminology we have *arbitrarily* assumed, and from {sic} the underlying assumptions ... of the equations from which the equilibrium is determined.' (D3/12/11: 67) At the time Sraffa was of the opinion that Petty's concept of 'food' – a proper

Finally, in early or mid 1928, Sraffa abandoned his previous view as to the non-payment of interest on seed and fixed capital. He saw that the premiss on which it rested was unfounded, namely, that this would not endanger the self-replacement of such capital items and, as a consequence, that of the system as a whole. Clearly, this premiss was contradicted by the 'self interest' of entrepreneurs: 'The real difficulty here is that, if it is a matter of inducement, we shall have to induce the wheat {seen as 'seed'} growers *not* to transfer their wheat in other investments' (D3/12/6: 10 and 12; similarly D3/12/11: 82a). And in a document in which he defined the analytical steps ahead in his work with reference to chapters I-III of part I of volume II of *Capital* he emphasized: 'necessità di interesse su cap. fisso: se no, non è possibile accumulaz. proporzionale in tutte le industrie – chi presterebbe a un'industria che non rende abbastanza da riprodursi? {necessity of interest on fixed capital: if not, proportional accumulation in all industries is not possible – who would lend to an industry which does not yield enough to reproduce itself?}' (D3/12/9: 11) As to the character of the surplus to be allotted to fixed capital, Sraffa stressed that since fixed capital cannot immediately be disinvested – or 'withdrawn' – in order for capitalists (as a class) to be able to consume more, 'Therefore, interest on fixed cap. is a rent.' (D3/12/9: 87)⁷

The Smithian definition of fixed capital, centred on the aspect of circulation, turned out to be not very useful. However, there is also a second line which started essentially from Ricardo whose focus had exclusively been the role of capital in production and, in particular, the

'physical real cost' – instead of labour – a 'metaphysical' concept – was the appropriate starting point of a coherently objectivist approach to the problem of value and distribution. See on this also Garegnani (2004).

⁷ The concept of withdrawing played an important role when in the late 1920s Sraffa attempted to 'objectivise' the existence of interest (and rent). While wages are considered a 'necessary' (or 'natural') cost of production, 'enabling' workers to subsist and perform their productive task, interest is seen as a 'social' cost incurred by society in order to prevent capitalists from decumulating their capital and consuming it unproductively. Since in the short run capitalists can only withdraw the circulating part of their capital (including the funds accumulating to make good the wear and tear of fixed capital), but not the fixed part, in order to ward off the threat of withdrawal interest has to be paid only on the former part.

degree of longevity of the different capital goods employed in production.⁸ As we shall see, this perspective was eventually adopted by Sraffa in place of the Smithian one. However, Sraffa did not agree with Ricardo that the distinction between fixed and circulating capital was neither important nor clear-cut. In the second edition of the *Principles* Ricardo had attached a footnote to his definition of fixed capital saying: 'A division not essential, and in which the line of demarcation cannot be accurately drawn.' (Ricardo 1951: 51 and 31 n; see, however, also the first edition, *ibid.*: 150). Similar views had been expressed, *inter alia*, by William Stanley Jevons ([1871] 1965: 242), Alfred Marshall ([1890] 1970: 63, n. 2) and Knut Wicksell ([1901] 1934: 184). In a piece written in November 1927 Sraffa objected:

It should not be understood that the distinction between fixed and circulating capital is not a sharp line, and that they shade into one another. The distinction is *not* based on a difference of things: we cannot make two catalogues, one of the things that enter into fixed, the other of the things that enter into circul. The distinction is this. We have a factory with machines, stock, food, at the beginning of the year. At the end of the year we have the product *plus* the residuums of fixed capital, depreciated: the house decayed, the machine worn, etc. Now, all that does not {sic} more exist was circulating cap.: all that survives outside the product is fixed. (D3/12/4: 1(1))

In this passage the distinction is decided in terms of the conventional classical concept of the length of the production period, that is, a year: fixed capital goods are accordingly those produced means of production whose economic life is longer than a year, whereas all other capital goods are circulating. This involves, of course, an element of arbitrariness which did not escape Sraffa's attention. In 1928, in the context of a discussion of the *reduction* of a commodity to dated quantities of 'food' in the without- and the with-surplus economy, he observed that in a circular flow scheme 'The production of a thing has no real definite beginning – the inquiry leads us into infinite time backwards.' (D3/12/7: 27)

3. Reducing fixed capital to circulating capital and land: 'looms'

⁸ Ricardo had specified: 'According as capital is rapidly perishable, and requires to be frequently reproduced, or is of slow consumption, it is classed under the heads of circulating, or of fixed capital.' (Ricardo 1951: 52)

There are numerous documents in Sraffa's papers, written at different times, all dealing with 'looms', with the first composed as early as late 1927 and early 1928. Obviously, over time the questions raised under this heading changed, reflecting the progress of Sraffa's analysis and his becoming aware of new problems and solutions. His respective discussions comprise basically all aspects related to the presence of durable instruments of production. They include, in particular:

- the problem of capital utilization, a typical example being the case of shift work;
- the possibility of expanding productive capacity by re-investing the accumulating annuities of ageing looms;
- the case of accumulation that follows a cyclical pattern or one of expansion and stagnation;
- the case of accumulation in the face of ongoing technical progress.

In the manuscripts on which we focus attention, Sraffa deals with the most elementary problem of fixed capital: its role in the theory of value and distribution of an economic system in a self-replacing state. The main issue Sraffa had to address was whether it was at all possible *vis-à-vis* durable instruments of production to preserve a purely *objectivist* approach in the theory of value and distribution. By the time of June 1928 he had managed, with the help of Frank Ramsey, to establish firmly that this was in fact possible in the case of circulating capital (see Kurz and Salvadori 2001: 262-4). Did this carry over also to the case of fixed capital? A similar question had already bothered the classical economists who had seen that durable instruments of production introduced a complication into the theory of value and distribution. While the circulating part of the capital advanced at the beginning of the period of production contributes entirely, at the cost of its bodily existence, to the annual output, that is, 'disappears' from the scene, the contribution of the durable part is less obvious and the idea of a material transmigration into the product or its total evaporation seems to lose any foundation: at the end of a production cycle a (partly worn out) machine is still there without much sign of physical degradation. How then to deal with fixed capital in the theory of value and distribution?

In the winter of 1927-28 Sraffa, assuming a 'social' point of view, sought to approach the problem under consideration not in terms of an analysis of single items of ageing fixed capital

but in terms of whole groups of such items characterized by a balanced age composition.⁹ His aim was to reduce such groups to circulating or working capital, on the one hand, and land (or perennial capital), on the other: whereas the former enters in its entirety into the product, the latter does not enter into it at all.^{10,11} Sraffa saw the following dichotomy:

Suppose that we have 100 looms: that each loom lasts 10 years¹², and that 10 are 10 years old, 10 are 9 years old ..., 10 are new. Average age 5 years. Now, at the end of the year we shall have scrapped 10 looms 10 years old,

⁹ The point of view of 'society as a whole' was also underlying Irving Fisher's discussion of 'looms' in *The Nature of Capital and Income* (Fisher 1906: 244-7). Sraffa annotated the respective pages in his copy of the book. With regard to a simple numerical example Fisher arrived at the conclusion: 'if the 20 looms are evenly distributed throughout the different stages of wear, and if, for convenience, we assume that one loom wears out each year, no depreciation fund will be necessary. The replacement of one loom annually is equivalent to such a depreciation fund, and the capital is thereby maintained at a constant level.' (Ibid.: 246)

¹⁰ This was in line with M. Pantaleoni's distinction between 'flow' and 'fund'; see Pantaleoni (1894: 363), a book which Sraffa knew well. He thus did not follow Walras and the Walrasians who had reduced all produced means of production to perennial capital – an everlasting fund of services similar to land. The reasons for Sraffa's dislike of this approach can easily be inferred from Pareto's following rationalisation: 'Since, by a fiction which brings us more or less close to reality ..., capital goods are deemed to remain always in their original condition, we cannot say that they are transformed into the product. It is their use alone which contributes toward obtaining this product, and we will say that it is the SERVICE of the capital which is transformed.' (Pareto [1927] 1971: 223; 1906: 289-90) See also Sraffa's annotations in the *Manuale* and his excerpts from it in D1/91: 62.

¹¹ It hardly needs to be stressed that the concept of 'reduction' here is different from the one used earlier, that is, the reduction to (dated) quantities of food or labour.

¹² In a note written in December 1927 he stressed that the life of machines is uncertain; what really matters is how the lives of machines scatter around an average; see D3/12/10: 64.

and all the others will have grown 1 year older. (We assume decay to be continuous and even, at rate of 10% a year). The total decay is equal to 10 new looms. During the year we will have had to use so much circ. cap. as required to make 10 new machines. Therefore of the original 100 10 (one per age) were circulating cap., and 90 were fixed.

He continued, still entertaining his early view of interest on fixed capital:

Now interest may be paid on the 10 circ. cap., but *must not* be paid on the 90 fixed (It must be paid to the industry: not necessarily on capital: the division with labor {sic} is arbitrary) [The above shows also that machinery receives an interest so long as it is a "product", or that it is "being consumed": later, nothing] (D3/12/4: 1(1-2))¹³

In a related manuscript written somewhat later he made clear that the 90 aged machines actually constituted fixed capital of an everlasting character and thus could analytically be identified with 'land':

The 10 new machines are entirely, in every particle, 100% working capital; they disappear from the world during one period of production, without leaving trace of themselves, except 10 entirely worn out machines which are no more machines but rubbish to be thrown away. The 90 old machines, of average age $4\frac{1}{2}$ years, are 100% fixed capital; not a bit of them is consumed, or worn out in production. They are there at the end of the year, in the exact state in which they were at the beginning, with exactly the same age composition and the same expectation of life – they have not grown any older, although a year has lapsed. As it were, they have been *mere spectators in production, they have "contributed" nothing, they have "transfused no part of themselves" into the produce.* (D3/12/5: 13; emphasis added)

He added with reference to Gonner's edition of Ricardo's *Principles*: 'The old machines are, to all effects, "land", viz. "undestructible" but not original' (D3/12/5: 13; see also 16 and D3/12/11: 97). The circulating part of social capital, including the circulating capital needed to make good the wear and tear of fixed capital, together with the output levels, were then taken

¹³ In the same document Sraffa expressed doubts as to his presupposition and allowed interest on fixed capital, stressing that it constitutes a quasi-rent. See also D3/12/5: 25.

by Sraffa as the material basis on which the social surplus was assessed and interest paid, whereas the land-like part was taken to play no role in this respect. Sraffa insisted that the entire capital stock of looms of different ages could either be reduced to circulating capital, comprising the brand-new items replacing the worn-out ones, or land, comprising all the aged items. He emphasized:

Depreciation, there is no such thing. Fixed capital is eternal: and working capital is entirely destroyed in one period of production. These two categories are exhaustive: *tertium non datur*. (D3/12/5:12-13)¹⁴

This was certainly an ingenious device to deal with fixed capital which, as Sraffa noted, could be traced back to Johann Heinrich von Thünen.¹⁵ It would have allowed Sraffa, or so he initially thought, to render the problem amenable to two pieces of economic theory already at his disposal at the time: (a) the theory of circulating capital in his first and second equations; and (b) the classical theory of differential rent as reformulated and rectified by him in the late 1920s. However, the approach was not without serious shortcomings the following of which deserve mention. First, as we have seen, it was explicitly designed in such a way that the question of how the prices (or book-values) of machines and thus their depreciation quotas were ascertained did not even arise; hence the value of the stock of fixed capital and the values of its various items could not be determined. Then there were the difficulties arising precisely from the fact that a perennial machine, while 'undestructible', is not 'original'. The difference matters since one cannot consider the 'scarcity' of a perennial machine in the same way as one considers the scarcity of land: while land is typically considered to be original and nonproduced, the amount of a perennial machine existing in a given economy depends on previous decisions and therefore cannot be considered scarce in the long run. As a consequence we cannot use the theory of extensive (or intensive) margins

¹⁴ See again Irving Fisher's remark that in such a case 'no depreciation fund will be necessary' (Fisher 1906: 246).

¹⁵ See his reference to Böhm-Bawerk (1890: 166n and 172n) and his comment: 'Von Thünen uses exactly the same method, as the 10 looms of 10 ages, for having capital "undestructible": he deals with tools or horses.' (D3/12/10: 37) Sraffa then appears to have studied the French translation of Thünen's *Der isolirte Staat*; see D3/12/9: 37.

to determine prices.¹⁶ Third, it was unclear whether the efficiency profile of a machine over its entire economic life had an impact on the general solution of the system as a whole. Fourth, there was no reason to presume that the approach could be generalised to cases that were less special than the one of a balanced age composition of durable capital in conditions of no technical progress. And finally, and most important: Was the approach really compatible with Sraffa's objectivist research programme?

Sraffa, in fact, appears to have quickly felt that he could not leave matters at that. To begin with, he attempted to make good the shortcoming or lacuna mentioned first by complementing his approach with what in his 1960 book he was to call the 'Accountant's method'. Indeed, already in the winter of 1927-28 we see Sraffa employ the well-known annuity formula which he had adopted from books such as Predella (1915, n. 183) and Baily (1808). Accordingly, the annual charge to be paid for interest and depreciation, a , amounts to

$$a = \frac{Cr(1+r)^n}{(1+r)^n - 1} \quad (\text{D3/12/9: 28})$$

where C is the value of the new machine, r the rate of interest and n the life of the machine. He added that for 10 machines a would be ten times the above amount. With the appropriate age distribution, each year a new machine has to be bought, and subtracting its value C from the formula for 10 machines, there remains what Sraffa called 'l'Annualità variabile':

$$\begin{aligned} a_1 &= \frac{10Cr(1+r)^n}{(1+r)^n - 1} - C \\ &= \frac{10Cr(1+r)^n - C(1+r)^n + C}{(1+r)^n - 1} = \frac{(10Cr - C)(1+r)^n + C}{(1+r)^n - 1} = C \frac{(10r - 1)(1+r)^n + 1}{(1+r)^n - 1} \end{aligned}$$

Sraffa expounded:

The last formula gives the perpetual *net* annuity that I get from fixed capital. If I divide it by r (i.e. I multiply it by the years' purchase) I get the capital value of fixed capital, i.e.: (substituting n for 10, since in my hypothesis they must be equal)

¹⁶ At that time Sraffa had not yet solved the problem of intensive rent, but he had already elaborated the theory of extensive rent within the chosen objectivist framework.

$$\text{Val. F.C.} = \frac{(nC - C)(1+r)^n + C}{r(1+r)^n - r} = \frac{nC(1+r)^n - \frac{C}{r}(1+r)^n + \frac{C}{r}}{r(1+r)^n - 1}$$

If I divide this by C , I get the relation between value of *new* machines and of *secondhand* machines. (ibid.: 28-9)

From this he calculated the value of the balanced stock of n machines as¹⁷

$$\frac{(nC - \frac{C}{r})(1+r)^n + \frac{C}{r}}{(1+r)^n - 1} \quad (\text{D3/12/9: 30})$$

In a paper titled 'Value of capital' (D3/12/7: 139-40) he also drew attention to the following implications. First, the value of fixed capital, expressed in terms of a new machine, 'varies with the rate of interest, it increases with a rise in r and decreases with a fall' (ibid.: 139). This finding made him rethink his view as to how much the proprietor of fixed capital can actually disinvest (i.e. 'withdraw') *vis-à-vis* a change in the rate of interest and thus a change both in the value of his capital and the income he can derive from it. Secondly, with increasing age the value of a fixed capital item (operating with constant efficiency) falls by equal steps in successive years *only* in the case of a zero rate of interest, whereas if the rate of interest is positive the downward steps increase with age (see also D3/12/9: 26). In fact, the jig-saw Figure 6 of his 1960 book is anticipated in several documents, including the one under consideration. Sraffa emphasized:

¹⁷ The procedure Sraffa followed is not the obvious one, but it is very ingenious to anyone who knows that the value of a perpetual rent at a constant rate equals the ratio of the rent rate over the rate of interest. The obvious procedure consists in finding the value of a machine of date t ($1 \leq t \leq n$) and summing them over t . An echo of this analysis is found in §§ 82-3 of Sraffa (1960) where, however, the formula for the value of the balanced stock of n machines is not given in general. On the contrary we find here the formula of the value of a machine of date t ($1 \leq t \leq n$). The reader may suppose that Sraffa is here following the obvious procedure. The value of the balanced stock of n machines is mentioned in the text and in the caption to Fig. 6. In particular the limiting values for $r = 0$ and for r tending to infinity are given. However the attentive reader can check that the value given for $r = 0$ is wrong ($n/2$ instead of $(n+1)/2$).

Notice however, that a change in the rate of interest means a total revolution in the values of commodities, including machines ...

How will the value of circulating capital vary with a variation in the rate of interest? (D3/12/7: 139-40)

It did not take Sraffa long to see that the problem could not satisfactorily be treated in this way. Clearly, the accountant's method was restricted to the special case of constant efficiency.¹⁸ More important, in a note written in early summer 1928 Sraffa denied that even in this case a solution was at all possible. He asked,

how is the value of the fixed capital determined? not by the equations, which are insufficient in number. If then this value is determined by some other method – e.g. their actuarial value or earning power (but this implies knowledge of r , which implies solution of the equations, which imply *previous* knowledge of the value of fixed capital) it might be determined so as to make it correspond to r .

No all this is impossible. (D3/12/9: 67)¹⁹

Now, since all previous efforts had drawn blank, in which direction to seek a solution next? One possibility was, of course, to adopt that approach which we eventually encounter in the 1960 book. Interestingly, Sraffa had come across early expressions of it already in the late 1920s. In the next section we deal briefly with the available evidence why at the time he rejected the joint-products approach.

4. Fixed capital as a joint product: Sraffa's view in the 1920s

Sraffa's papers and annotations in his books document that he had encountered the joint-products approach to fixed capital several times via the literature he consulted in the initial period of his constructive work, and even before. For example, he noted in his own index of

¹⁸ On the probable origin of the concept of 'efficiency' of fixed capital, see D3/12/10: 26.

¹⁹ Interestingly, N. Georgescu-Roegen made an attempt to revive the fund-concept of capital in the 1960s, that is, after the publication of Sraffa's 1960 book. For a criticism of it, see Kurz and Salvadori (2003).

his working copy of Marshall's *Principles*: 'fixed capital accounted for as produce 172'. On the page mentioned Marshall indeed counterposes two methods of dealing with fixed capital, the second being the joint-products method. He exemplifies the latter in terms of production on a farm:

Sometimes it is convenient to speak as though *all the capital applied were circulating capital* applied at the beginning of the year or during it: and in that case everything that is on the farm at the end of the year is part of the produce. Thus, young cattle are regarded as a sort of raw material which is worked up in the course of time into fat cattle ready for the butcher. The farm implements may even be treated in the same way, their value at the beginning of the year being taken as so much circulating capital applied to the farm, and at the end of the year as so much produce. This plan enables us to avoid a good deal of repetition of conditioning clauses as to depreciation, etc., and to save the use of words in many ways. It is often the best plan for general reasonings of an abstract character, particularly if they are expressed in a mathematical form. (Marshall 1922: 172; emphasis added)

As we know, reducing fixed capital to circulating capital was the approach Sraffa eventually adopted. He came across the joint-products method also in volume VI of the French edition of *Theorien über den Mehrwert* in a passage in which Marx refers to Ricardo's view that what is not consumed of fixed capital is reckoned as part of the product (see Marx, 1925, vol. VI: 110, and Sraffa's own index at the end of the book), and then in winter 1927-28 apparently when reading the French edition of volume I of *Das Kapital* (Marx 1900). In a note written as a response to a passage in the posthumous second edition of Malthus's *Principles of Political Economy*²⁰, quoted by Marx in *Capital*, vol. I (1954, p. 205 n.), Sraffa commented:

This passage is quoted, with approval by Marx Cap. I, 91, and his context shows that he adopts the same point of view (see also Hist., VI, 110. "la partie non consommée du capital fixe est comptée dans le produit {the non consumed part of fixed capital is reckoned among the product}"). Marx

²⁰ 'If we reckon the value of the fixed capital employed as a part of the advances, we must reckon the remaining value of such capital at the end of the year as a part of the annual returns.' (Malthus 1836: 269)

allowed therefore himself to be misled by Malthus! But since he says that "sous le nom de capital constant avancé ... nous ne comprenons donc jamais que la valeur des moyens consommés dans le cours de la production {under the name of advanced constant capital we never understand anything else than the value of the means consumed in the course of production}" (Cap. I, 91) his conclusions may not be affected by it. (D3/12/10: 52)

At the time Sraffa appears to have thought that the joint-products method, by blurring the concept of product, also blurred the concepts of physical real cost and thus of surplus. He therefore did not adopt the method himself. This is perhaps also the reason why he almost ignored it when he came across it again on the occasion of copiously excerpting from the third edition of James Mill's *Elements of Political Economy* (1826) around May 1932. The passage that interests us here, and which Sraffa copied, is the following one:

The case of fixed and {of} circulating capital may be treated as the same, by merely considering the fixed capital as a product, which is regularly consumed and replaced, by every course of productive operations. The capital, not consumed, may be always taken, as an additional commodity, the result of the productive process. (D3/12/9: 111; emphasis added. Cp. Mill (1826: 80-81))

5. From the late 1920s to the early 1940s

From the summer of 1928 to the beginning of 1931, when he interrupted his work for a decade, Sraffa basically continued to think of fixed capital as reducible to circulating capital and land. However, there are clear signs of growing doubts as to the sustainability of this view. This may be exemplified by a manuscript of altogether 18 pages entitled 'Looms' (D3/12/13: 16 (1-18)) composed in the summer of 1929.

The paper opens with the claim that Marshall's 'quasi-rents' are actually a mixture of pure rent and pure interest which ought strictly to be kept apart: 'we must give the formula for this division' (ibid.: 16 (1)). Next Sraffa expressed anew his original view: 'Analogously we must show that every piece of fixed capital is made up of two parts, one of which is perfectly mobile capital (or circulating, or liquid), the other perfect land.' However, the germ of its abandonment is clearly visible in the following self-criticism:

The error I have made in previous attempts to determine to what extent the owner of the looms can disinvest and transfer his capital (having in mind consumption, not "abstaining") when the income he derives from them falls, is this: that I assumed the income from the looms to vary *with* (in correspondence, but not in proportion to) the general rate of interest. This must *not* be done.' (Ibid.: 16 (1))²¹

Sraffa had to admit that the impact of the level of the rate of interest, which affects the length of the 'period of production'²² and thus impinges on the result that obtains, could not be ruled out. This undermined the basic premiss of the entire exercise, namely that there is a clear-cut distinction between fixed capital respectively seen as circulating capital and as land:

A notable point is that thus the distinction between "land" and "circ. capital" ceases to be absolute (i.e. determinable by mechanical or chemical properties, or by mere length of abstract time) but becomes variable with the rate of interest. ...

The original assumption was "perfect mobility of capital" and "perfect immobility" of land. This appears to be an abstract postulate, not realized in practice ... In reality, it was now realized, there are only intermediate cases, of greater or smaller mobility; "rent is but the leading species of a large genus", and thus Marshall introduces the concept of Quasi-rent. We now see however that this is not a new category ... Every element in this category can be divided in two parts ... of which one belongs purely to one extreme, the other purely to the other extreme ... We thus return to the original postulate. *But* whereas then the categories had to be embodied in distinct persons, objects or places, and became blurred themselves if the embodying thing became indistinct, *now they seem to lead an independent existence in the clouds of abstraction.* (Ibid.: 16 (5-6); emphases added)

²¹ Sraffa put a straight line in the margin of this passage.

²² At the time Sraffa tried to describe production processes with the help of the Austrian concept of 'period of production' although he was clear that the latter could not be defined independently of the level of the rate of interest.

Sraffa admitted that "'looms" are not a fair parallel to "land"'. Instead, he surmised, 'We ought to take the whole of fixed capital'; yet, he felt, 'this we cannot do, because there is no sharp distinction from circulating' (ibid.: 16(11)). Indeed, the dependence of which part of fixed capital is 'land' and which is 'circulating capital' on the rate of interest undermines the absolute character of the dichotomy. Deprived of it the whole basis of the concept of 'withdrawing' and all that follows from it falls to the ground.

While Sraffa still tried to uphold the conceptualization with which he had started his probing steps into the theory of fixed capital, there were now considerable cracks in the mirror. Later in the paper he also stressed that his previous emphasis on the 'income of the capitalist' (which could be used for consumptive purposes and involve disinvestment of fixed capital) was mistaken and that 'the "rate of interest" is a much more general fact':

The rate of interest enters in the determination of the length of the period of production, of the sinking fund and depreciation of instruments of production, of the value of commodities; all things which are not more related to the income of the capitalist than to other incomes. (Ibid.: 16(14))

Sraffa was clearly not content with what he had achieved up till then. The deficiencies of the reduction of fixed capital to circulating capital and land were all too obvious. But was there a compelling alternative to it? In a paper of four pages titled 'Fixed capital', dated 7 and 15 November 1931, with an insertion on 17 November, he stated with great clarity the problems he faced and his ideas about how to solve them. The paper reflects the analytical framework mentioned in the above and the corresponding concepts, such as the 'period of production', and discerns as the main problem of fixed capital the fact that it gives rise to a *sequence of outputs over time*, or what was later called intertemporal joint production. The paper documents neatly the difficulties Sraffa faced and the remedies by means of which he intended to overcome them. His concern with measuring the various kinds of machinery only 'in *physical* units' is clearly expressed. Alas, Sraffa could not realize the analytical steps indicated in the paper under discussion because the first period of his constructive work was about to come to a close.

He resumed his constructive work at the earliest convenience after his release from the internment camp on the Isle of Man in October 1940. In the camp he had studied carefully the recently published new English edition of volume I of *Das Kapital* (Marx 1938). It appears to have come somewhat as a surprise to Sraffa to see to what extent Marx had been wrestling with the same or similar problems. Apparently Sraffa was impressed by Marx's conceptual

and analytical achievements, relative to where Ricardo had left the subject and in how far he had anticipated elements of Sraffa's own analysis. Back in Cambridge Sraffa studied volumes II and III of *Das Kapital*, using the French edition (Marx, 1901, 1902), while also consulting the original German and the English. His respective readings are reflected in numerous references to Marx's works and ideas and in the adoption of some of Marx's concepts. In particular, Sraffa took over the concepts of 'constant' and 'variable capital', 'organic composition of capital', and 'period of rotation', a term hinting at his use of the French edition.²³

Another important source of some of Sraffa's respective work carried out in mid August of 1942 was the English edition of volume I of Knut Wicksell's *Föreläsningar i Nationalekonomi*, published in 1934 (Wicksell 1934). In an appendix the volume reproduced Wicksell's discussion of Gustaf Åkerman's attempt to replace Böhm-Bawerk's concept of the 'average period of production' with the technical lifetime of a fixed capital item as a measure of capital intensity, where the lifetime itself was to be determined by cost-minimising producers ('Åkerman's problem').

At around this time Sraffa also began his collaboration with the mathematician Abram S. Besicovitch whom he was to thank 'for invaluable mathematical help over many years' (Sraffa 1960: vi; see also Kurz and Salvadori 2004a). The models Sraffa developed with Besicovitch's assistance reflect several lines of his thought, most of which had originated in the earlier period of his constructive work. The most important one was the following. After having developed his first and second equations Sraffa developed his 'third equations', that is a system with a surplus and variable wages as analysed in sections 8-11 of chapter II of the book. He thus turned to the problem of how relative prices and the rate of interest change with changes in the real wage rate, given the system of production. After some deliberations he convinced himself in a manuscript composed in February and March 1931 that in a circular flow framework

it may be said that the value of total capital in terms of total goods produced cannot vary, since the goods are composed exactly in the same proportions as the capitals which have produced them.

²³ The English translation of the German 'Umschlagsperiode' (or 'Umschlagszeit') is 'period of turnover'.

He added though:

Both propositions are false, but may contain an element of truth. (D3/12/7: 157 (3); emphasis added)

It was not immediately clear whether the 'element of truth', whatever it was, carried over to the case with fixed capital. As we have seen in the above, by that time Sraffa had already established the fact that with a rise in the rate of interest the value of a stock of machines of uniform age composition expressed in terms of a brand-new item of machinery tends to increase. Does this imply that the value of a set of machines would also rise with respect to that of the social product as the rate of interest increases? And if so, would this increase in the value of fixed capital be exactly offset by a contrary movement of the value of circulating capital such that the ratio of capital as a whole to product would remain constant, that is, independent of income distribution, as Sraffa's propositions required?

It was especially this question which attracted Sraffa's attention from August 1942. We now encounter his old proposition under the name of 'My Hypothesis' or simply 'Hypothesis'.²⁴ There was a second element that played an important role in this context. Already towards the end of the previous period Sraffa had started to distinguish between an 'Agricultural' and an 'Industrial scheme' of production. While according to the former production is envisaged as a discontinuous process which evolves in yearly cycles from seed to harvest, according to the latter it is essentially a continuous process that could be started (and finished) at any moment of time. With regard to industrial production it was therefore close at hand for Sraffa to have recourse to Knut Wicksell's continuous time analysis. In order to investigate whether his Hypothesis holds in such a framework, Sraffa in collaboration with Besicovitch in August 1942 elaborated a number of simple models featuring both circulating and fixed capital (see

²⁴ For a formulation that comes close to the 1931 one, see the document dated 'Jan. 43' titled 'New Models': D3/12/32: 1-2. It had also not escaped Sraffa's attention that a similar proposition had been put forward by Marx in terms of a given organic composition of capital describing the technical conditions of production of the economy as a whole via the ratio of constant capital (C) to variable capital plus surplus value ($V+S$), or 'dead' to 'living' labour. It was the discovery that Marx had essentially adopted the same point of view and had elaborated similar concepts that prompted Sraffa to develop his own argument henceforth in close touch with Marx's, a feature not present in the first period of his constructive work.

D3/12/20: 27-47, D3/12/17 and D3/12/19). One such model has two sectors (machines and bread), with machines whose duration is n years produced in an 'Austrian' process by unaided labour, and bread produced by labour and machines. Another model which Sraffa dubbed the 'port model' was apparently inspired by Wicksell's famous discussion of the maturing of wine in the *Lectures* (Wicksell 1934: 172-84). However, Sraffa understood swiftly that this model did not cover the important case of machinery. He stressed:

The trouble when applied to machinery is that, when r {the rate of interest} rises, there is nothing to prevent any old machine becoming more valuable than a new one: this is OK for port, but absurd for machines. (D3/12/20: 44)

September 1942 sees Sraffa engage in 'Attempts at Fixed Capital' (D3/12/21). The starting point of his investigation was a view entertained by Marx in volume III of *Das Kapital* which, in Sraffa's translation from the German edition, reads:

As regards the ratio of the composition of the capitals out of fixed and circulating capital, taken in itself and by itself, it affects the rate of profit not at all. (D3/12/21: 79; see also Marx 1959: 151-2)²⁵

Marx added that this ratio can only indirectly affect the rate of profit, via different ratios of variable to constant capital or different periods of turnover during which a given profit will be realised. Sraffa devoted a long manuscript to the study of the two influences. In this context he then reformulated the distinction between circulating and fixed capital:

We distinguish units of input into two classes:

- a) those which can be followed during their transit through production and traced on a given unit of output. These form Circ. Capital
 - b) those which cannot be so traced, because they are used in the production of a succession of units of output and only gradually wear out. These Fixed Cap.
- (D3/12/21: 49)

The fact that fixed capital units cannot be 'traced' on a given unit of output turned out to be the source of much trouble. The idea of a material transmigration of an input into a

²⁵ Sraffa's concern with Marx's proposition is understandable not least because of Sraffa's earlier view on the matter in which fixed capital had no role to play with regard to the allotment of the surplus (and thus the determination of the rate of interest).

corresponding output looked obsolete and prompted Sraffa to contemplate ways out of the impasse. One of these consisted in 'measuring the time each unit of input takes to come out in the product (incl. fixed capital): then average, weighting by values.' Alas, the difficulties also of this method were all too 'obvious':

what is the value of the capital stock? it cannot be found as a list of materials; for it consists, not of a definite number of separate articles (as commodities are), but of a continuous series of objects in course of transformation. Besides, it would involve setting up equations for their valuation (as for commodities), and although this would be an absurdly artificial procedure, as these things (half-worn machines, burning coal, undeveloped photos, etc.) have no market. – Finally this would represent the capital stock as one of the fundamental quantities, which is the source of many fallacies. Throughout this we assume that the capital stock does not appear explicitly in the system. (Ibid.: 54)

The situation did not look all that promising. It was aggravated by two facts both of which Sraffa had been able to establish with Besicovitch's help. First, the models with fixed capital had shown that the Hypothesis did not hold in general even if all capital is circulating (ibid.: 42). Since the actual system could not be relied upon to satisfy his Hypothesis, Sraffa in December 1942 felt prompted to seek a solution in terms of an artificial device, a *Hilfskonstruktion*:

We must therefore produce a *construction* that will show, evidently, that the value of the commod. on the left h.s. in terms of those on the r.h.s. {of the equations} is constant with respect to r ; which is not at all obvious if Fixed Cap. appears in the form $\frac{r(1+r)^n}{(1+r)^n - 1}$. (D3/12/27: 32; emphasis added)

It goes without saying that Sraffa's respective efforts resulted in the construction of the *Standard system*. Secondly, it was not possible in general to reduce all fixed capital to dated quantities of labour because the reduction series exhibits alternating positive and negative elements (see D3/12/21: 25).²⁶

²⁶ In his 1960 book we shall read: 'This will make it impossible on the one hand for the residual aggregate of commodities to tend to vanishing-point and on the other for the sum of labour terms to tend to a limit.' (Sraffa 1960: 67-8)

6. Reducing fixed capital to circulating

It was in the winter of 1942-43 that Sraffa finally achieved a break-through to which we now turn. The two folders under consideration are, respectively, D3/12/27, composed from October to early December 1942, and D3/12/30, composed between 10 and 28 December of the same year. The former bears the title 'Fixed Cap. (Equivalent Circ.)', the latter 'Fix.Capital – Final'. In the former Sraffa was keen to reach full clarity about what the problems were and how they could be tackled. He no longer followed the idea that had guided him during the first period of his constructive work, that is, of reducing fixed capital to circulating capital and land. The revised perspective, and the stage of maturity of his thoughts, are well expressed in a paper of seven pages dated 'Oct. 1942' and titled 'Fixed Capital Equivalent Circulating' (D3/12/27: 46). When at a much later time Sraffa brought order to his old notes and manuscripts he qualified this document as containing 'some fairly vivid points' on 'why we are mystified by Fixed Capital and must try to reduce it to Circulating' (ibid.: 47). Sraffa took up the thread where he had dropped it in November 1931. He stated the difficulties posed by durable instruments of production with even greater lucidity than in the papers referred to in the preceding section and emphasized that they can only be overcome by reducing them to circulating capital. It is apposite to quote large parts of the paper:

The difference between Circulating and Fixed Capital is this. A piece of circulating capital (such as a lump of raw material or fuel, or a day's labour), after a transit of a definite duration in the process of production, comes out all at one moment and transfers all its value to a piece of product ready for sale. A piece of fixed capital (such as a building, or a machine, or a working animal) on the other hand gives out its value over a period of time, to a whole series of pieces of product which are successively turned out ready for sale.²⁷

When $r = 0$, and commodities exchange at their Values²⁸, the matter is clear. A machine which lasts n years (without repairs and with equal

²⁷ In the margin of the passage Sraffa added: 'Also longer period of rotation.'

²⁸ The reference is to labour values.

efficiency) transfers to the product of each year of its life $\frac{1}{n}$ th of its original value. It is at once apparent that the machine can be regarded as equivalent to n pieces of circulating capital, each being equal in value to $\frac{1}{n}$ th of the machine, but each having a different period of rotation; one piece having a period of one year, one of 2 years, etc., and one of n years.²⁹ But when there is a rate of profit, the matter becomes confused. Then, the owner of the machine must still receive a uniform annual payment in respect of the machine (for profit and depreciation together); for if he did not, and the payment of successive years were different, the products of the machine would have different prices according as they were produced by an old or a young machine. And yet, since the book-value of the machine falls with age, the profit must fall, and depreciation increase. And this difference in annual depreciation (or what part of the original cost is transferred to the product) will vary with the rate of profit. ...

He went on:

The difficulty arises from this: for circulating capital, at the same moment that its value passes into the product, in most cases, also the material substance which is the bearer of that value, either passes into the product (raw material) or anyway passes out of the process of production (e.g. fuel). On the other hand, for fixed capital, the transfer of value from, e.g., the machine to the product, appears as a purely abstract process, which takes place without any corresponding transfer of material substance: that value is passed is undoubted, for the machine decreases in value while the product increases, but the machine remains complete in all its parts, with its efficiency unimpaired for the time being, and ready to resume operation in the next year.³⁰

In order to see how this abstract process takes place an abstract point of view is inevitable.

²⁹ § 80 of Sraffa (1960) can be traced back to the above passage.

³⁰ In the margin of this passage Sraffa put a straight line.

We must first notice that when the rate of profit rises a piece of circ. cap. adds more to the price of the product, in the shape of additional profit upon itself. But that is not all: the circ. cap. itself changes in prices (rises or falls) and this is an additional source of change (+ or -) in the addition to the price of the product.

The same happens for fixed cap. While the annual capital charge *increases*³¹ with r , the price of the new machine also changes, and thus the charge is changed. But the machine has one more degree of freedom than the circ. cap. While the latter must multiply every part of its value by $(1+r)$, as that is the only profit factor which it has, the machine has a whole range of choice. For it must multiply the part of its own price which it transfers to the product of the first year of its own life, by $(1+r)$; the part transferred to the product of the second year by $(1+r)^2$; etc.; and the part transferred to the product of its last year by $(1+r)^n$.

But this process of transfer is purely abstract: if the machine transferred to the product of each particular year a particular part of itself (e.g. the pistons to that of the first year, the cylinders to that of the second etc.), there would be no option: it would have to transfer the value of the particular part in the particular year. But nothing of the kind happens: the transfer is purely abstract; and while the machine is bound to transfer parts equal in magnitude of value each year, it is free to choose *which* particular part it will transfer in any one year. For these parts of value though equal in magnitude, need not be equal in kind: the machine has been produced by a certain quantity of commodities and by a certain quantity of labour, and accordingly it derives its value in part from the former and in part from the latter.

And on 17 October he added:

Why do we want to reduce Fixed Cap. to Circulating Capital?

Because we must prove that *the only quantities whose knowledge is required to determine prices are the Capital Inputs; while the Capital Stock is not required, and anyhow is only a derived quantity, derived from the inputs.* (D3/12/27: 46 (1-5); the last emphasis is ours.)

³¹ In Sraffa's manuscript the word has a wavy underlining.

On 11 November 1942 Sraffa specified a number of assumptions whose implications he was about to investigate:

We shall assume that a fixed capital 1) lasts for a given number of years, 2) during which it operates with equal efficiency and 3) without need of maintenance, repairs, or replacement of parts.

1) It is clear that fixed capitals will die a natural death at different ages, just as men do. We assume that they all live to the average age, as given in their mortality tables.

2) The fall in efficiency can be dealt with since it can be reduced to one of these two cases (or a suitable mixture of the two): the fixed capital can become less productive because a) it produces the same amount by using more labour, or fuel, or materials, and b) it produces less quantity by using the same labour etc. The annual price of the fixed capital can be adjusted by deducting from it, in case a) the price of the additional labour etc., and in case b) the price of the lost product [both of course appearing in the equations as unknowns, to be determined by the general solution].

3) Consider a motor-lorry that lasts 10 years, but requires new tires every year, and a new engine after 5 years. We regard the tires as a fixed capital that lasts a year, the engine as another fixed capital that lasts 5 years, and the rest of the lorry, without tires or engine, as a fixed capital that lasts ten years: thus segregated, each of the items lasts its own lifetime without need of replacement. (D3/12/27: 38)

A few days later he put the following question to Besicovitch:

How to divide p_m into n parts x_1, x_2, \dots, x_n such that

$$x_0 = x_1(1+r) = x_2(1+r)^2 = \dots = x_n(1+r)^n$$

and also $x_1 + x_2 + \dots + x_n = p_m$ {?}

Clearly, p_m is the price of the new machine; $x_t(1+r)^t$ is the the annual charge on the machine during year t (which, whatever is t , must be the same since the efficiency is constant, otherwise the price of the product would depend on the age of the machine which is used to produce it), x_t is the abstract portion of the machine which is transferred to the product during the t -th year of its utilization. Obviously the price of the t year old machine, p_{mt} , can be determined as

$$p_{mt} = \left[p_m - \sum_{h=1}^t x_h \right] (1+r)^t.$$

With the problem well stated, Besicovitch had no difficulty in finding the solution. In the same document we read in his hand:

$$\begin{aligned} x_n &= x_n \\ x_{n-1} &= x_n(1+r) \\ x_{n-2} &= x_n(1+r)^2 \\ &\dots\dots\dots \\ x_1 &= x_n(1+r)^{n-1} \\ x_1 + \dots + x_n &= x_n \frac{(1+r)^n - 1}{r} = p_m \\ x_n &= \frac{r p_m}{(1+r)^n - 1}. \end{aligned} \tag{D3/12/27: 6}$$

This problem was solved, but the 'purely abstract' character of the involved process of value transfer was hardly able to put Sraffa's mind at rest. In fact, on 4 December 1942 he jotted down a short piece titled 'Fixed into Circ. – *Objective basis*'. We quote it in full:

(After division of original price into proportional parts which pass into the product in successive years).

It may now be asked, can the matter be left at that? Certainly not. For it is against the object of the present treatment which is to represent the production and circulation of commodities in material terms (i.e. quantities of labour, of commodities and periods of time) *independent of* the distribution of the product, i.e. of the rate of profit.

This can be seen at once if we try to apply Reduction. How much labour, and of what period enters into a commodity. The answer would be, it *depends* on r . If we thus make the quantity of labour entering a commodity depend on r , we are falling straight into the Böhm-Bawerk-Wicksell blunder of making the period of production depend on the rate of interest. We also open the way to another danger; there is nothing to stop that method being applied to the Reduction of wages (as being composed of a quantity of commodities that varies with r) and even to profits. Which would reduce the whole scheme to nonsense.

Therefore it is necessary to make the transformation of Fix. into Circ. in terms, *not* of proportionate parts varying with r , but of actual quantities of labour and commodities, of such magnitudes and of such periods that they will *happen* to vary in price (and not in quantity) as those proportionate parts.

(The trouble of this division into actual quantities of labour and commodities is that it is so arbitrary, and does not correspond to any real events (even if it is possible to do it in *every* case, i.e. for all compositions of capital).)

To this objection against the "abstract" division, as making the *quantities* of labour and commodities *dependent* on r it might be replied: this is true only if we take an isolated machine (individual point of view): but if we take a complete group of machines (social point of view) it is no longer true. (D3/12/27: 11-12)

And four days later, in a document bearing the same title, he specified what he meant by 'objective' in the case under discussion as 'giving for each instalment quantities of labour and commodity which are independent of r ' (D3/12/27: 8 (1)).

Sraffa also wanted to understand better how his method compared with that of the accountant as put forward in textbooks of commercial arithmetic.³² The latter method he had already studied in the late 1920s. There is a paper of some fifteen pages on this issue written between 2 and 6 December 1942 (D3/12/27: 21-35). Besicovitch does not seem to have directly been involved. We encounter a Sraffa who is now sure that the questions he had raised were right on target:

There are innumerable schemes of dividing the fixed annuity of Fixed Cap. between replacement of capital and profit. *All* these schemes are equally correct if they satisfy the requirements:

1) That the total paid in each successive year is constant.

³² An early source Sraffa consulted was Baily (1808), a book used at the London Stock Exchange. When he worked on the accountant's method in December 1942 his main source appears to have been Kent (1927); see, especially, D3/12/27: 21-30.

2) That the total paid for replacement of capital at any time is equal (*without* the addition of any interest) to the original cost of the asset.³³
(D3/12/27: 33)

In an earlier part of this manuscript (written on 2 December 1942) Sraffa mentioned explicitly the joint-products method but at the time was of the opinion that this would not be of much use because 'there is no equation to determine the value of the one-year older machine on the right {hand side of the equation}: this must therefore be determined separately, and for this the formula giving the annuity for Fixed Capital is required.' (D3/12/27: 31) A few days later, on 10 December, Sraffa adopted for good the joint products-approach to fixed capital. He specified:

If in the production of a commodity there enters a fixed capital (e.g. a machine) which has a working life of m years, then we divide the industry that produces it, into m sections each of which uses machines of only one of the m possible ages, and for each section we set up an equation. On the l.h.s. there will be the materials etc., as well as a number of machines of, say, i years' age, and on the right hand side the quantity of commodity produced by the section, plus a number of machines of $i + 1$ years' age: except for the section using $m - 1$ years old machines, which on the r.h.s. will have only the commodity, and no machines – or at most the scrap, the price of which must already be determined elsewhere as a raw material / in other branches of production which produce it and use it as a r.{aw} m.{aterial}.

We thus have, for each commodity produced by a fixed capital, in addition to the single original equation, $m - 1$ new equations, and on the other hand the prices of $m - 1$ age-groups of machines to be determined: apart from the equation of the industry that produces the machine, to which corresponds its price when new.

If the machines have a fixed term of life, at the end of which they are discarded, each section will employ the same number of them. But if, as is in general the case (and necessarily so if the f.c. consists of working animals etc.), they are worn out gradually each section will employ the number which, according to the mortality tables of the f.c. in question, are

³³ There are two straight lines in the margin of point 2).

required to maintain the population stationary when the number used in the section which employs new machines are added (or born) every year.

The machines of different ages may have differing efficiency, and also require varying quantities of labour and materials for their maintenance and repair: these differences will be shown in the equations of the respective sections, in varying quantities appearing on the l.h.s. and in differences in the product.

Calling the fixed capital f , and the quantity of it used new in the production of commodity A, F_a , its price p_f ; the number of machines 1 year old ${}_1F_a$, and their price p_{f1} etc., denoting the machines of ages 1, 2, ..., $m - 1$ years and the sections using them respectively by the suffixes 1, 2, ..., $m - 1$ we write the equations for commodity A³⁴

$$(A_{a1}p_a + \dots + F_a p_f + K_{a1}p_k + L_{a1}w)(1+r) = A_1 p_a + {}_1F_a p_{f1}$$

$$(A_{a2}p_a + \dots + {}_1F_a p_{f1} + K_{a2}p_k + L_{a2}w)(1+r) = A_2 p_a + {}_2F_a p_{f2}$$

.....
 $(A_{am-1}p_a + \dots + {}_{m-1}F_a p_{f_{m-1}} + K_{am}p_k + L_{am}w)(1+r) = A_m p_a$

If we assume that the machines have a fixed term of life, during each year of which their efficiency and maintenance are constant and the scrap value nought, we can verify that their prices at various ages which are the solutions of the equations agree with the book-values ascribed to them by the "natural" plan of depreciation which is recommended for those conditions. (D3/12/30: 5-8 and 10)³⁵

On the following day Sraffa continued the manuscript dealing now with the problem of whether the 'Reduction method' was applicable to the case under consideration. His conclusion was negative, echoing an earlier finding of his and Besicovitch's.³⁶ He saw

³⁴ We corrected some obvious typos in the second equation.

³⁵ These considerations eventually became §§ 74 and 76 in Sraffa (1960).

³⁶ It could be applied only in the case of rising efficiency.

that the products of Fix. Cap. are inextricably mixed up as joint products [here they appear as products of "successive years" of the same machine] and there is no sense (not merely no possibility) in allotting fractions of cost (of capitals) to different parts of them. Each atom of Fix. Cap. appears to go into the whole series of products: there is no way (indep. of r , of course) of dividing them up. (D3/12/30: 13)

In the remaining days of December 1942 he investigated the solvability of the above equations and whether in the case of constant efficiency the accountant's method gives the same result as the joint-products method.

On 27 July 1943 Sraffa returned to the problem of fixed capital in order to round off his respective argument: the reference is to a paper titled 'Two methods of reckoning fixed: "Malthus's" and "Accountant's"' (D3/12/34: 12). He stressed that the former regards 'old machines as part of annual product' and that both methods 'conform to the rule of equal annual decay' (ibid.), which is the rule stated in the document of 6 December 1942. Sraffa had finally managed to solve the problem of fixed capital in terms of 'Malthus's method' which in the late 1920s he had rejected as inadequate. The method made its way into the published book where it is dealt with in §§ 73-74. However, in Appendix D, 'References to the Literature', the method is now traced back to Robert Torrens, from whom Ricardo, Malthus and Marx appear to have borrowed it. There Sraffa provides evidence that Torrens (1818, p. 336; 1821, p. 28) had anticipated the idea and therefore deserves to be credited with it.

The comparison between the two methods in §§ 75-78 culminates in the proof that the two are equivalent in a single case only, that is the only case in which the accountant's method applies: that of constant efficiency throughout the life of a machine. This equivalence is proved in § 76. The proof was found with the help of Besicovitch. On 20 and 21 August 1943 Sraffa and Besicovitch each contributed bits to a piece titled '*Fixed Capital – The Two Methods (Deducing one from the other)*' (D3/12/34: 1-2). The document starts in Sraffa's hand:

As we have two methods for dealing with Fixed Capital ((a) the Accountant's method and b) Malthus's method of treating aged machines as products) which give identical results, they must be deducible from one another.

This brings the collaboration between Sraffa and Besicovitch on fixed capital to a close.³⁸ In late summer 1943 the material we eventually encounter in Chapter X of *Production of Commodities by Means of Commodities* was ready, waiting to be put in a publishable form.

7. Conclusion

The paper provides a short history of the path Sraffa followed in his investigation of the problem of fixed capital. Two time spans are covered. The first extends from November 1927 until 1931, the second period from summer 1942 until the completion of his analysis of fixed capital in 1943. While initially Sraffa had advocated a concept of fixed capital that was similar to Adam Smith's, focusing attention on whether a commodity enters into exchange, calling fixed capital those commodities that don't, he soon abandoned it in favour of one that was more akin to Ricardo's and focused on production.

Sraffa identified the problem of fixed capital to consist in the following: while the circulating part of capital contributes entirely to the annual output and may be envisaged to transfer its value together with its material substance to the product, the contribution of the durable part is less obvious and the idea of a material transmigration into the product seems to lose any foundation. At first Sraffa thought it possible to retain his objectivist approach by reducing fixed capital to circulating capital on the one hand and perennial capital or land on the other. With regard to a capital stock consisting of 'looms' exhibiting a balanced age composition, the new looms entering production each year are considered circulating capital, whereas the old looms are 'mere spectators in production' that do not grow any older – just like land.

The approach had several shortcomings. To overcome the impasse, Sraffa in the late 1920s had recourse to the 'Accountant's Method' of dealing with durable instruments of production. Alas, this method was only compatible with the case of constant efficiency. More important, it presumed that the value of fixed capital was known, which, however, was an unknown. Interestingly, at the time Sraffa rejected the joint products-approach early expressions of which he had encountered in the writings of Marshall, Marx and Malthus. The difficulties besetting Sraffa's own approach came to the fore when in August and September

³⁸ We set aside a few other documents which relate to the problem of fixed capital but are of little interest.

1942 he wanted to find out whether a proposition he had put forward in early 1931 with regard to only circulating capital carried over to systems with fixed capital. This proposition – 'My Hypothesis' as he variously called it – stated that the ratio of the value of the net product to that of (non-wage) capital was independent of the rate of interest. With his mathematical friend Besicovitch he elaborated a number of models to investigate this problem. He found out that the Hypothesis could not generally be sustained and that the method of reduction to dated quantities of labour could not generally be extended to cover also the case of durable instruments of production. What was worse from Sraffa's point of view was that the case of fixed capital seemed to stand in the way of developing a purely objectivist theory of value and distribution.

However, in the winter of 1942-1943 Sraffa finally managed to overcome all his previous difficulties in this regard by reducing fixed capital to circulating in terms of the joint products-method. With the help of Besicovitch he then swiftly succeeded in formalising the problem of fixed capital in a satisfactory manner and determine book values and depreciation quotas both for the case of constant and non-constant efficiency profiles. Sraffa had thus accomplished an important task in his overall intellectual enterprise and had succeeded in showing that an objectivist view of the production and circulation of commodities was possible. He had accomplished it some two years before he became acquainted with John von Neumann's paper on equi-proportionate growth (von Neumann 1945 [1937]) when Nicholas Kaldor, then editor of the *Review of Economic Studies*, asked David Champernowne to write a commentary on the paper to be published alongside with an English translation of it.³⁹ Hence the view occasionally encountered in the literature that Sraffa's analysis of fixed capital may have been inspired by von Neumann's contribution finds no support from Sraffa's papers.

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³⁹ On von Neumann's paper, Champernowne's commentary and the help he received from Sraffa in interpreting the paper, see Kurz and Salvadori (1993, 2004b).

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