# 'Classical' vs. 'Neoclassical' Theories of Value and Distribution and the Long-period Method*

By Heinz D. Kurz and Neri Salvadori

## 1. Introduction

In this paper we ask whether classifying economic theories in distinct analytical approaches to certain economic problems and even in different schools of economic thought is a futile enterprise. More particularly, we shall argue that there is a thing that may, for good reasons, be called 'classical' economics, which is distinct from other kinds of economics, in particular 'neoclassical' economics. We shall focus attention on the theory of value and distribution. What we have in mind is a particular rational reconstruction of 'classical' economics, which, in our view, is both useful for an understanding of certain important arguments found in several classical authors and for an understanding of important present-day problems.

The paper consists essentially of two parts. The first part (Sections 2-5), responding to a request of one of the organisers of the conference, Professor Fabio Petri, is historical and provides a bird's eye view of the developments in the theory of value and distribution that took place since the inception of systematic economic analysis at the time of the classical economists. The second part (Section 6) instead is designed to demonstrate the power of the long-period method elaborated by the classical economists in terms of two examples. More specicfically, we begin, in Section 2, with a brief account of alternative answers to the question raised by this paper. Next, in Section 3, we turn to a discussion of the complexity of most economic problems and of economic theory as an attempt to come to grips with that complexity. This leads us to the identification of a first characteristic feature of classical economics: its *long-period* method. As we shall see in Section 4, a version of this method was also shared by all major neoclassical authors until the late 1920s. However, the similarity of the methods adopted by two theories

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must not be mistaken for a similarity of the content of the theories. This aspect becomes clear when we turn, in Sections 3 and 4, to the scope and content of traditional classical and traditional neoclassical economics. The emphasis is on the sets of data, or independent variables, on the basis of which these theories attempt to explain the respective unknowns, or dependent variables, under consideration. It will be seen that in this regard classical economics differs markedly from neoclassical economics, the main difference being the way in which income distribution is determined. Sections 3 and 4 also raise the question whether the sets of data contemplated by these theories are compatible with the long-period method or whether there exist tensions and contradictions between method and content of a theory. It is argued that whilst traditional classical theory can be formulated in a consistent way, traditional neoclassical theory faces insurmountable difficulties in this regard. The latter come to the fore in the shape of inconsistencies that undermine the logical foundation of the approach to the problem of income distribution in terms of the demand for and the supply of the factors of production collaborating in the generation of the social product, when there are produced means of production, i.e. 'capital', among these factors. Section 5 turns to the attempts of neoclassical authors from the late 1920s onwards to remedy this defect and at the same time render the theory more 'realistic', and indeed 'dynamic', in terms of models of temporary and intertemporal equilibria. It can be argued, however, that these alternatives are beset by a number of methodological difficulties. Moreover, it is close at hand to ask the following question within the neoclassical framework: Are there problems that can fairly easily be grasped using the long-period method, whereas these problems are very difficult to deal with in an intertemporal analysis? Section 6 exemplifies the power and fecundity of the long-period method in terms of two special problems: the first problem concerns a multisector variant of the AK growth model, the steady-state properties of which can be analysed without invoking some bold assumptions needed in an intertemporal model; the second concerns the determination of one of the distributive variables and relative prices in the empirically important case in which fixed capital items are jointly used.

## 2. A brief account of alternative views of 'classical' economics

The view that the economics of the classical authors from Adam Smith to David Ricardo is essentially just an early variant of neoclassical theory, that is, an explanation of quantities and relative prices, including the prices of factor services, in terms of demand and supply functions, was advocated by Alfred Marshall. According to him classical analysis is characterized by a fairly well developed supply side, whereas the demand side is still in its infancy. Marshall's interpretation has been widely accepted in the history of economic thought and was shown to be untenable only relatively recently by Piero Sraffa with his reconstruction of the development of

Ricardo’s theory of value and distribution in his introduction to volume I of *The Collected Works and Correspondence of David Ricardo* (Sraffa, 1951) and Sraffa’s reformulation of the classical approach to the theory of value and distribution in his book *Production of Commodities by Means of Commodities* (Sraffa, 1960). For a while it looked as if Sraffa’s alternative interpretation was about to be generally accepted. Yet from the mid 1970s things began to change somewhat. It was particularly due to contributions by John R. Hicks and Samuel Hollander that a version of Marshall’s integrationist perspective on the history of economic thought – also known as the 'continuity thesis' – gained momentum again. Amongst historians of economic thought the main spokesman of this point of view is Samuel Hollander according to whom all major economic theorists, including Smith, Ricardo and Marx, were demand and supply theorists of sorts. Hence from the point of view under consideration there is no such thing as a distinct classical approach to the theory of value and distribution.

This point of view has been taken to receive some support with regard to Sraffa’s 1960 reformulation of the classical theory of value and distribution by attempts to show that Sraffa’s analysis is but a ‘special case’ of intertemporal general equilibrium theory. This view was put forward, for example, by Christopher Bliss (1974), but it was particularly Frank Hahn (1982), who gave prominence to it. Many economists today appear to share some version of the view that there is essentially a continuity of ideas from the very inception of systematic economic thought beginning with the time of the classical economists. According to this view the history of economic theory can essentially be conceived of as a one-way avenue leading from primitive conceptualizations of the demand and supply approach to all sorts of economic phenomena to ever more sophisticated ones, merely leaving behind errors of reasoning and unnecessarily restrictive assumptions.

However, there are exceptions to the rule even within the group of contemporary authors who are major proponents of one version or another of neoclassical theory. One such exception appears to be Kenneth Arrow who in a paper on Ricardo’s theory of distribution (Arrow, 1991) and also in a paper co-authored with Starret (Arrow and Starret, 1973) stated clearly that Ricardo’s theory defies of being subsumed under neoclassical theory, because of a different analytical structure, in which demand functions play no role in the determination of the general rate of profits and the real wage rate. The latter is rather taken as given, when determining the profit rate and relative prices. (On Arrow's view see especially Garegnani, 2000.)

More than a century ago, around the time of the so-called 'marginalist revolution'[1], the perception that the theories advanced by Jevons, Menger and Walras constituted a fundamental break with the classical tradition was even more pronounced. Jevons stressed that he once and for all wanted to do away with the 'mazy and preposterous assumptions' of the classical economists (Jevons, [1871[ 1965, p. xiii); in addition, he accused them of having put forward a theory that is indeterminate: He wrote:

> 'Another part of the current doctrines of Economics determines the rate of profit of capitalists in a very simple manner. The whole produce of industry must be divided into the portions paid as rent, ... profits and wages. ... Rent also may be eliminated, for it is essentially variable, and is reduced to zero in the case of the poorest land cultivated. We thus arrive at the simple equation –
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> Produce = profit + wages.
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> A plain result also is drawn from the formula; for we are told that if wages rise profits must fall, and *vice versâ*. But such a doctrine is radically fallacious; *it involves the attempt to determine two unknown quantities from one equation*. I grant that if the produce be a fixed amount, then if wages rise profits must fall, and *vice versâ*. Something might perhaps be made of this doctrine if Ricardo's theory of a natural rate of wages, that which is just sufficient to support the labourer, held true. But I altogether question the existence of any such rate' ([1871], 1911, pp. 268-9; emphasis in the original).

The accusation of having provided an indeterminate system recurred, inter alia, in the writings of Walras (([1874] 1954, § 368) and Wicksteed (1894). In contradistinction, Wicksell with his typical acuteness stressed that 'the way in which Ricardo develops his argument ... is a model of strictly logical reasoning about a subject which seems, at first glance, to admit of so little precision'; and 'Ricardo's theory of value is, one finds, developed with a high degree of consistency and strictness' (Wicksell [1893] 1954, pp. 34 and 40). He added: 'Since, according to Ricardo, wages represent a magnitude fixed from the beginning, and since – as he later shows – the level of rent is also determined by independent causes, the cause of capital profit is already

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1 As we shall see, the theories that filtered into the profession in the final third of last century constituted no revolution at all because all main building blocks of these theories had already been forged in the five decades between Ricardo's death (1823) and the publication of the works of Jevons (1871), Menger (1871) and Walras (1874).

settled. It is neither possible nor necessary to explain capital profit in other ways, if the other assumptions are sound' ([1893] 1954, pp. 36-7). Therefore, in Wicksell's view Ricardo's system was not underdetermined. This did not mean, of course, that Wicksell agreed with the content of Ricardo's theory; it only meant that he was willing to admit that there was a classical alternative to the then already conventional demand and supply approach, which he, Wicksell, was engaged in further elaborating by way of integrating the Austrian, essentially Böhm-Bawerkian, temporal view of production and consumption into a Walrasian general equilibrium framework.

Although Wicksell and some other commentators were prepared to concede that there was a distinct classical or Ricardian theory of value and distribution, the analytical structure of this theory was far from clear to most economists. In fact, one of the reasons for the abandonment of the classical approach to the theory of value and distribution was that it was considered to be unable to accomplish the task it had set itself, that is, to determine the rate of profits and relative prices in a logically consistent manner. Another reason for the attacks on and eventually abandonment of that theory was, of course, the use to which it, and especially the labour value-based reasoning, had been put by the so-called Ricardian socialists. The classical approach was close to falling into oblivion and a proper understanding of it vanished. It was only after a long period of time that the classical approach was rediscovered and its analytical structure gradually laid bare from under thick layers of interpretation (see below).

So what are the characteristic features of the classical approach to the theory of value and distribution?

## 3. The classical approach to the theory of value and distribution

The concern of the classical economists from Adam Smith to David Ricardo was the laws governing the emerging capitalist economy, characterized by wage labour, an increasingly sophisticated division of labour, the coordination of economic activities via a system of interdependent markets in which transactions are mediated through money, and continuing technical, organisational and institutional change. In short, they were concerned with an economic system in motion. The attention focused on the factors affecting the pace with which capital accumulates and the economy expands and how the growing social product is shared out between the different classes of society: workers, capitalists and landowners.

*Long-period method*

How to analyse such a highly complex system? The ingenious device of the classical authors to see through these complexities and intricacies consisted of distinguishing between *market* or *actual* values of the relevant variables, in particular the prices of commodities and the rates of remuneration of primary inputs (labour and land), on the one hand, and *natural* or *normal* values, on the other. The former were taken to reflect all kinds of influences, many of an accidental and temporary nature, whereas the latter were conceived of as expressing the persistent, non-accidental and non-temporary forces governing the economic system. The classical authors did not consider the 'normal' values of the variables as purely ideal or theoretical; they saw them rather as 'centres of gravitation' of actual or market values (cf. Smith, *WN*, I.vii). This assumed gravitation of market values around their natural levels was seen to be the result of the self-seeking behaviour of agents and especially of the profit-seeking actions of producers. In conditions of *free competition*, that is, the absence of significant and lasting barriers to entry in and exit from all markets – the case with which the classical authors were primarily concerned – this involved *cost minimization*. This was well understood by the authors under consideration, and hence their attention focused on what may be called *cost-minimizing systems of production* (see also Kurz and Salvadori, 1995, esp. chs 1 and 13).

The method of analysis adopted by the classical economists is known as *long-period method* or method of *long-period positions* of the economy. Any such position is nothing but the constellation towards which the system is taken to gravitate, given the fundamental forces at work in the particular situation under consideration. In conditions of free competition the resulting long-period position is characterized by a *uniform rate of profits* (subject perhaps to persistent inter-industry differentials), *uniform rates of remuneration* for each particular kind of primary input in the production process (such as different kinds of labour and natural resources), and prices that are assumed not to change between the beginning of the uniform period of production and its end, that is, *static prices*. Such a constellation is to be understood as reflecting the salient features of a competitive capitalist economy in an ideal way: it expresses the pure logic of the relationship between relative prices and income distribution in such an economic system. The prices are taken to allow producers to cover just costs of production at the normal levels of the distributive variables, including profits at the ordinary rate. These prices have aptly been called also *prices of production* (Torrens, Ricardo).

Ever since the advent of systematic economic analysis in the 17th and 18th centuries, economists have aspired to elaborate a proper dynamical theory and many ingenious and hard-working people have made great efforts in this regard. However, given the complexity of the object of their analyses – a socioeconomic system incessantly in travail – they understood that the long-period method was the best available to them. The latter proved indeed quickly to be a powerful tool for studying certain properties of complex interdependent systems; that is, systems which it would be extremely difficult to model and analyze in a dynamic framework, even with the advanced tools of modern mathematics at one's disposal. Moreover, the classicals themselves occasionally ventured probing steps in the direction of such a dynamical analysis. Think, for example, of David Ricardo's discussion of the introduction and diffusion of improved machinery in the newly added chapter 'On Machinery' in the third edition of his *Principles*, published in 1821 (cf. *Works*, I, ch. 31). However, a general dynamic analysis of the highly complex system under consideration, paying due attention to all relevant interdependencies, was not considered possible in principle, and it is doubtful that had they considered such an analysis possible, they would have considered it of much use. The long-period method was envisaged as the best available in order to come to grips with the basic driving forces, and their interplay, of an ever-changing world characterized by continuing technical progress, the depletion of natural resources and a changing distribution of income. Long-period analysis was precisely devised to overcome the impasse in which the social scientist found himself, confronted with a reality which, at first sight, looked impenetrable, made up of a myriad of relationships between people amongst themselves and people and natural objects. The long-period method gave some transparency to the complex object of study and allowed the theorist to derive a large number of interesting insights into the functioning (and the sources of malfunctioning) of the economic system. Because of its fecundity, the long-period method was almost generally adopted in political economy until the 1930s and still plays an important role in contemporary economics.

This does not mean that there was no interest in short-period problems amongst the classical economists; there was, of course. However, the important point to be made is that, in the majority of authors dealing with such problems, the short-period analyses elaborated by them had – as their backbones, so to speak – fully specified long-period theories. In other words, the long-period theory was considered the core of economic analysis from which they derived several short-period analyses designed to tackle special problems of a short-run nature, such as the implications of a capital stock that is not fully adjusted to the other data of the system or a sudden increase of the quantity of money in circulation.

*Characteristic features of the classical approach*

It is a first characteristic feature of the classical economists' approach to the problem of value and distribution that the 'data' or rather independent variables contemplated all refer to magnitudes that can, in principle, be observed, measured or calculated. This point of view, which may be called 'objectivist', is clearly expressed, for example, in William Petty's *Political Arithmetick* and in the physiocrats, in particular in François Quesnay's *Tableau Économique*. It is also present in the writings of Adam Smith, David Ricardo and Karl Marx: these authors refrained from having recourse to any magnitudes that are non-observable, non-measurable or non-calculable in determining the general rate of profits and relative prices.

Secondly, the many differences between different authors notwithstanding, the contributions to the theory of value and distribution of 'classical' derivation typically start from the same set of data. In general, these concern the following:

(1) The set of technical alternatives from which cost-minimizing producers can choose.
(2) The size and composition of the social product, reflecting the needs and wants of the members of the different classes of society and the requirements of reproduction and capital accumulation.
(3) The ruling real wage rate(s) (or, alternatively, the rate of profits).
(4) The quantities of different qualities of land available and the known stocks of depletable resources, such as mineral deposits.

The treatment of wages (or, alternatively, the rate of profits) as an independent variable and of the other distributive variables, the rate of profits (the wage rate) in particular, as dependent residuals exhibits a fundamental *asymmetry* in the classical approach to the theory of value and distribution. In correspondence with the underlying long-period competitive position of the economy, the capital stock is assumed to be fully adjusted to these data, especially to the given levels of output. Hence the 'normal' desired pattern of utilization of plant and equipment would be realized and a uniform rate of return on its supply price obtained. Prices of production are considered the medium of distributing the *social surplus* in the form of profits between different sectors of the economy and thus different employments of capital and, with scarce natural resources, in the form of differential rents of lands and mines.

It deserves to be emphasized that these data are sufficient to determine the unknowns, that is, the rate of profits (the wage rate), the rent rates, and the set of relative prices supporting the cost-minimizing sytem of producing the given levels of output. No other data, such as, for example, demand functions for commodities and factors of production, are needed. The classical approach allows a consistent determination of the variables under consideration: it accomplishes the task put to itself. It does so by separating the determination of income distribution and prices from that of quantities, taken as given in (2) above. The latter were considered as determined in another part of the theory, that is, the analysis of capital accumulation, structural change and socioeconomic development. More precisely, the magnitudes referred to in the set of 'data' (1)-(4) are only treated as known or given in one part of classical theory: the determination of the shares of income other than wages, and relative prices, in given conditions of the economy. In other parts of the theory they are themselves treated as dependent variables or unknowns. Hence, variables (1)-(4), while magnitudes external to the classical approach to the theory of value and distribution in particular, are magnitudes internal to the classical theory as a whole. This draws attention to the fact that the classical authors distinguished between different spheres of economic analysis necessitating the employment of different methods. While one sphere is suited to the application of deductive reasoning – this relates to the investigation of the relations between the distributive variables and relative prices, given the system of production – the other sphere requires more inductive lines of reasoning and research – this relates to an investigation of the sources and consequences of economic change, in particular technological progress, economic growth, changing consumption patterns, the exhaustion of natural resources etc.

*Modern classical economics*

It hardly needs to be stressed that with the above specification of 'classical' political economy, this school of thought did not vanish with the death of Ricardo or some other early classical economist. Constituting a fertile research programme, the classical approach managed to survive during the two centuries since its inception, albeit with several ups and downs. The danger of extinction was repeatedly warded off by scholars who, after decades during which the classical approach had been 'submerged and forgotten' (Sraffa), managed to lay bare again its genuine significance and clarify its characteristic features.

Early formalisations of the classical approach to the problem of value and distribution were provided, among others, by Vladimir K. Dmitriev and Ladislaus von Bortkiewicz. John von Neumann contributed to the classical approach in terms of his famous model of economic growth. However, there is one author in particular whose work is uniquely important for the revival of classical political economy: Piero Sraffa (1951, 1960).[2] He deserves credit both for his work in the history of economic thought, tracing the classical approach back to David Ricardo and before him the physiocrats, and his coherent reformulation of that approach. Sraffa's work entailed a renewed interest in the writings of the old classical authors and induced analytical contributions leading to a host of new findings concerning complex economic systems characterized by the production of commodities by means of commodities.

Independently from Sraffa's work there were contributions, mainly in the tradition of the von Neumann model, with a strong classical flavour. These concerned, for example, the so called 'non-substitution' and 'turnpike theorems'. The non-substitution theorem states that, under certain

2 For a recent critical assessment of Sraffa's contributions to economics, see Kurz (2000) and Cozzi and Marchionatti (2000).

specified conditions (one primary factor, no joint production and constant returns to scale), and taking the rate of profits (rate of interest) as given from outside the system, relative prices are independent of the pattern of final demand. The theorem was received with some astonishment by authors working in the neoclassical tradition since it seemed flatly to contradict the importance attached to consumer preferences for the determination of relative prices. As Samuelson wrote: 'From technology and the interest rate alone, *and completely without regard to the demand considerations* ...[,] price relations can be accurately predicted as constants' (1966, p. 530; emphasis in original). In the usual Marshallian price-quantity diagram in order for demand to exert an influence on the price of a good, the supply function must not be horizontal. It is the hypothesis that the rate of profits (or, alternatively, the wage rate) is given and independent of the level and composition of output which accounts for the theorem. This hypothesis is completely extraneous to the neoclassical approach in competitive conditions and in fact assumes away the role played by one set of data from which that analysis commonly begins: given initial endowments (at most one could have a given amount of labour which, however, would not matter for the issue under consideration because returns to scale are constant: it would determine the size of unemployment because employment is determined by the other data). The assumption of a given rate of profits radically transforms the substance of that theory. With the endowment side chopped off, the concept of 'scarcity' of factors of production loses the significance usually attributed to it in neoclassical explanations of relative prices (there is no relative scarcity of primary and non primary factors). Hence the demand for goods, and thus preferences, can no longer exert an influence on prices via the derived demand for factor services which are available in given supply: prices of goods are independent of demand because income distribution is *assumed* to be independent of demand. It goes without saying that, in the framework of classical analysis with its different approach to the theory of value and distribution, a characteristic feature of which is the non-symmetric treatment of the distributive variables, there is nothing unusual or exceptional about the non-substitution theorem. A similar argument could be developed with respect to turnpike theorems.

## 4. Traditional neoclassical theory

*Long-period equilibrium*

The appeal exerted by the long-period method can be inferred from the fact that all early major marginalist authors, including William Stanley Jevons, Léon Walras, Eugen von Böhm-Bawerk, Alfred Marshall, Knut Wicksell, and John Bates Clark, fundamentally adopted it. Like the classical economists and Marx they were concerned with explaining the normal rate of profits and normal prices: the concept of long-period 'equilibrium' is the neoclassical adaptation of the classical concept of normal positions (see Garegnani, 1976). For example, in Marshall's *Principles of Economics* it is stated:

> The actual value at any time, the market value as it is often called, is often more influenced by passing events, and by causes whose action is fitful and short lived, than by those which work persistently. But in long periods these fitful and irregular causes in large measure efface one another's influence so that in the long run persistent causes dominate value completely (Marshall, [1890] 1977, p. 291).

And Böhm-Bawerk suggested that the investigation of the permanent effects of changes in what are considered the dominant forces shaping the economy should be carried out by means of comparisons between long-period equilibria. These comparisons are taken to express the 'principal movement' entailed by a variation in the basic data of the economic system (cf. Böhm-Bawerk, [1889] 1959, vol. 2, p. 380). This view was shared by Ludwig von Mises, one of the most radical subjectivists of the Austrian school of economic thought, who advocated the long-period method, or, as he preferred to call it, the 'static method', in the following terms:

> One must not commit the error of believing that the static method can only be used to explain the stationary state of an economy, which, by the way, does not and never can exist in real life; and that the moving and changing economy can only be dealt with in terms of a dynamic theory. *The static method is a method which is aimed at studying changes*; it is designed to investigate the consequences of a change in *one* datum in an otherwise unchanged system. This is a procedure which we cannot dispense with (Mises, 1933, p. 117; emphasis added).

However, the adoption of the long-period method was not, by itself, prejudicial as to the *content* of the theory. In order to see this we have to turn to the forces which the neoclassical approach, in contradistinction to the classical one, conceptualized in order to determine normal income distribution and the corresponding system of relative prices.

*The neoclassical set of data*

Since the new theory was to be an alternative to the classical theory, it had to be an alternative theory about the same thing, in particular the normal rate of profits and normal prices. However, the set of data in terms of which the neoclassical approach attempted to determine these variables exhibits some striking differences with respect to the classical approach. First, it introduced independent variables, that is, explanatory factors, that were not directly observable, such as marginal utilities and agents' preferences. Secondly, it took as given not only the amounts of natural resources available, but also the 'initial endowment' of 'capital'. The data from which neoclassical theory typically begins its reasoning are:

(i) The set of technical alternatives from which cost-minimizing producers can choose.
(ii) The preferences of consumers.
(iii) The initial endowments of the economy with all 'factors of production', including 'capital', and the distribution of property rights among individual agents.

The basic novelty of the new theory consisted of the following. While the received classical approach conceived the real wage as determined prior to profits and rents, in the neoclassical approach all kinds of income were explained simultaneously and *symmetrically* in terms of the forces of demand and supply with regard to the services of the respective factors of production: labour, 'capital' and land. This was obtained by invoking substitution mechanisms acting symmetrically on labour, land and capital from technical and consumption choices. The neoclassical theory was able to elaborate *functional relationships* between the quantity demanded, or supplied, of a service (or good) on one side and the price of that service (or good), and eventually other prices, on the other. It was the seemingly coherent foundation of such functional relationships that greatly contributed to the rapid success of neoclassical theory in economics.

Historically long-period neoclassical theory derives from a generalization of the theory of rent in terms of land of uniform quality and 'intensive' margins to all factors of production, including 'capital' (see Bharadwaj, 1978). This generalization presupposes that there is an analogy between land, labour and 'capital'. On this premiss the principle of scarcity rent, which the classical economists had limited to natural resources in given supply, was thought to be applicable also in explaining the incomes of labour and 'capital', that is, wages and profits. However, in order to be able to conceive of the rate of profits as some kind of index expressing the relative scarcity of a factor called 'capital', that factor had to be assumed to be available in a given 'quantity'. The degree of (relative) scarcity of the given 'quantity of capital', which was taken to be reflected in the level of the rate of profits, was then envisaged to be the result of the interplay of data (i)-(iii). The smaller the overall amount of capital at the disposal of producers, other things being equal, the greater in general the relative scarcity of that factor and the higher the rate of profits, and vice versa.

As regards the conceptualization of the 'capital' endowment of the economy, the advocates of long-period neoclassical theory, with the exception of Walras (at least until the fourth edition of the *Eléments*), were aware of the following fact. Whereas different kinds of labour and land can be measured in terms of their own physical units, 'capital', conceived of as a bundle of heterogeneous produced means of production, had to be expressed in terms of a *single magnitude*, related in a known way to the *value* of capital goods, allowing 'capital' to assume the physical composition or 'form' best suited to the other data of the system. For, if the capital endowment were to be given in kind, only a short-period equilibrium, characterized by differential rates of return on the supply prices of the various capital goods, could be established by the forces constituting demand and supply. Such an equilibrium could not, however, be considered a 'full equilibrium' (Hicks, 1932, p. 20). Whereas differential wage and rent rates for different qualities of labour and land are perfectly compatible with a long-period competitive equilibrium, differential profit rates are not: competition would enforce a tendency towards a uniform rate of profits. The discovery of *reverse capital deepening* and of the *reswitching of techniques*, that is, a technique is cost-minimizing at two disconnected ranges of the wage rate and not so in between these ranges, revealed the deficiency of the conventional neoclassical view: a central element of the explanation of distribution in terms of demand and supply – the principle of factor substitution as envisaged by the theory – cannot generally be sustained and, in particular, it cannot be applied to "capital".[3]

## 5. Temporary and intertemporal equilibrium theory

As early as the late 1920s some major protagonists of the demand and supply approach, especially Erik Lindahl, began to glimpse the deficiency of the conventional concept of 'quantity of capital' in an explanation of normal income distribution. However, confronted with the alternative of abandoning the demand and supply approach or the long-period method, in terms of which the former had been conceptualized, authors such as Lindahl, Friedrich August von Hayek and John Richard Hicks opted for the second alternative. The result of these attempts to overcome the impasse in which neoclassical theory then found itself was the development of the concepts of *intertemporal* and *temporary equilibrium*. In this way the demand and supply approach was meant to be rendered not only consistent but also more 'realistic' (cf. Lindahl, [1929] 1939, p. 271; Hicks, [1939] 1946, p. 116). Indeed, as the protagonists of the new developments kept stressing, economic theory had to be liberated from the straitjacket of 'static' analysis and turned into a proper 'dynamic' analysis.

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3 For a summary statement of the different versions of the theory and the debates around them, see Kurz (1987), Garegnani (1990a) and Kurz and Salvadori (1995, chap. 14).

This quest for greater 'realism' of economic theory could only have been strengthened by the observation that traditional demand and supply theory was not merely turning a blind eye to the complications posed by time, but was also logically inconsistent. Amongst the three Lindahl was perhaps best aware of this inconsistency. In a footnote added to the English text of his 1929 paper he pointed out that the received versions of 'modern' capital theory 'have the disadvantage that the measure of capital is made dependent on the prices of the services invested and on the rate of interest - which belong to the unknown factors of the problem' (Lindahl, [1929] 1939, p. 317).

Following Wicksell, Lindahl was concerned with incorporating the insights of the Austrian theory of capital into the time structure of production in a Walrasian theory of general equilibrium. He proceeded in terms of a sequence of models designed to exhibit rising degrees of 'realism'. This sequence was eventually to be crowned by a model capable of portraying, in abstract terms, a 'real' economy moving through time. It goes without saying that Lindahl did not achieve this bold aim and openly admitted this.[4] He was, however, convinced that the 'dynamic' approach to economic problems advocated by him liberated economic theory from a static dead end and shunted it on to the right track.

Lindahl himself did not use the term 'intertemporal equilibrium'; yet what he had developed in some parts of his analysis were clearly intertemporal equilibrium models with a finite time horizon. Since these were explicitly based on the assumption of perfect foresight they could represent no more than preliminary steps on the way to a 'general dynamic theory' that truly deserved this name.[5] Hence, while it is true, as Debreu (1959, p. 35, n. 2) observed, that Lindahl

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4 Towards the end of his paper Lindahl (ibid., p. 348) wrote that his investigation has been brought to a point 'at which a further approximation to reality is associated with ... considerable difficulties' which he, at the time, felt unable to tackle in a theoretically satisfactory way. These difficulties derived largely from the need to accomodate imperfect foresight and uncertainty in the model. His disenchantment with the achievements of his 1929 paper were also the main impetus for him to write the 1939 paper on 'The Dynamic Approach to Economic Theory' (Lindahl, 1939, part I).

5 Repeatedly, Lindahl expressed his uneasiness with the assumption of perfect foresight; see Lindahl (ibid., pp. 285 and 339-40).

provided 'the first general mathematical study' of this sort, its author can most certainly not be accused of having attributed too much importance to it.

In this conceptualization, in accordance with Walras's analysis, the capital stock available at a particular point in time, which is the beginning of the first period of the economy contemplated by Lindahl's theory, is given in kind. Hence: 'Produced capital goods have the same significance for price formation as true *original* sources of similar kinds' (ibid., pp. 320-1; emphasis added). The importance of the initial conditions for the dynamic behaviour of the economy is particularly stressed by Lindahl in his later paper: 'The first step in this analysis is *to explain a certain development as a result of certain given conditions* prevailing at the beginning of the period studied. These given conditions must be stated in such a way that they contain *in nuce* the whole subsequent development' (Lindahl, 1939, p. 21).

According to Lindahl the main feature distinguishing an intertemporal from a long-period equilibrium, which he identified with a 'stationary state', concerns the 'original' factors, including the capital goods in given supply at the beginning of the first period: 'The real difference from the stationary case lies in the circumstance that the primary factors, there regarded as given, are assumed to undergo change from one period to another. In this way a movement arises in the system" (Lindahl, [1929] 1939, p. 330). This movement concerns prices and quantities. As regards prices, Lindahl characterized the new 'dynamic' view of the economic system as opposed to the old 'static' one as follows: 'while in the stationary case the prices in succeeding periods are equal to the prices in the present period and thus do not introduce any new unknowns into the problem, in the dynamic case they will differ more or less from the prices in the first period' (ibid., p. 319). Correspondingly, the notion of a uniform rate of interest turns out to be generally devoid of any 'clear and precise content' (ibid., p. 245).[6] Nevertheless, Lindahl did not think that 'static theory' was entirely useless. He maintained rather that the system, if not disrupted by exogenous shocks, would gradually converge to a long-period equilibrium: 'If this tendency were alone operative, the community would in time reach stationary conditions' (ibid., p. 331), characterized by a uniform rate of interest throughout the economy.[7]

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6 See also the following statement by Koopmans: 'The irrelevance-in-principle of the concept of interest to the problem of efficient allocations over time is clearly implied, if not explicitly stated, in Lindahl's penetrating exposition of capital theory' (Koopmans, 1957, p. 114)

7 It is interesting to note the parallel between the view expressed by Lindahl and that of Duménil and Lévy (1985) in their response to Hahn (1982).

Similarly to Lindahl, Hicks, who was strongly influenced by Lindahl, emphasized that 'static' theory neglected important features of the 'real world', such as uncertainty and expectations, and thus was 'quite incompetent to deal properly with capital and interest' (Hicks, [1939] 1946, p. 116). 'Static theory', Hicks argued, would be applicable, 'if we could say that the system of prices existing at any moment depends upon the preferences and resources existing at that moment and upon nothing else.' Yet this is not the case: 'supplies (and ultimately demands too) are governed by expected prices quite as much as by current prices' (ibid., pp. 115-6). In his view the economic system had to be conceived of, 'not merely as a network of interdependent markets, but as a process in time' (ibid., p. 116). This process, he contended, was best represented as a sequence of temporary equilibria, each temporary equilibrium being dependent on individuals' expectations of the future. Just as in Lindahl, the productive equipment of the economy is assumed to be given in kind. While 'the economic problem' was traditionally conceived of, in an atemporal way, as consisting in the allocation of given ressources to alternative ends, it now had to be specified explicitly as involving 'the allotment of these resources, inherited from the past, among the satisfaction of present wants and future wants' (ibid., p. 130). In such a framework prices are bound to change. While own rates of interest can be defined, they are said to be 'of little direct importance for us' (ibid., p. 142).

Hicks's break with traditional neoclassical theory was even more radical than Lindahl's. The concept of the 'stationary state' as a position towards which the system is taken to gravitate if not perturbated by a series of exogenous factors of a more or less short-lived nature is rejected on the grounds that 'the stationary state is, in the end, nothing but an evasion' (ibid., p. 117).[8]

For quite some time Hayek's part in the development of the notion of intertemporal equilibrium had not received the attention it deserves.[9] One year prior to Lindahl Hayek had published a paper, in German, which for the first time bore the notion 'intertemporal equilibrium' (*intertemporales Gleichgewicht*) in its title. Hayek argued that contrary to the received opinion the existence of such 'equilibria' is not merely 'incompatible with the idea that constant prices are a prerequisite to an undisturbed economic process, but is in the strictest opposition to it' (Hayek,

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8 It is interesting to notice that Hicks later in his career became increasingly sceptical as to the usefulness of the temporary equilibrium method; see, for example, Hicks (1965, p. 66). On Hicks's recantation of the temporary equilibrium method, see Petri (1991).

9 It was only recently that his contribution to this field was given proper credit (cf. Milgate, 1979, and Huth, 1989).

1928, p. 37). Implicit in an intertemporal price system is a multiplicity of commodity-own-rates of interest (ibid., p. 43).

*Gérard Debreu*

A total break with the traditional method of analysis and its concern with long-period positions of the economic system characterized by a uniform rate of interest (profit) was finally effectuated in the so-called Arrow-Debreu model, developed by Kenneth Arrow and Gerard Debreu in the 1950s (Arrow and Debreu, 1954). Here we focus attention on Debreu's *Theory of Value*, published in 1959.

Debreu set himself two tasks: '(1) the explanation of the prices of commodities resulting from the interaction of the agents in a private ownership economy through markets, (2) the explanation of the role of prices in an optimal state of the economy' (Debreu, 1959, p. ix). Our concern will be exclusively with task (1). An 'economy' is defined in terms of three sets of data: (i) a given number of consumers, characterized by their consumption sets and their preferences; (ii) a given number of producers, characterized by their production sets; and (iii) total resources (cf. ibid., p. 74). As regards the latter, Debreu specified: 'They include the capital of the economy at the present instant, i.e., all the land, buildings, mineral deposits, equipment, inventories of goods, ... now existing and available to the agents of the economy. All these are a legacy of the past; they are *a priori* given' (ibid., p. 75). The property rights as to these resources are also taken as given; all resources are owned by consumers.

The abandonment of any concern with the long period in Debreu's analysis is also clearly involved in the assumptions of a given and constant number of producers and given shares of the profit of the various producers received by consumers (ibid., pp. 39 and 78).[10] As is well known, in Marshall the assumption of a given number of firms was entertained only in the short run, but not in the long run. Consequently, in the short run the supply function for any commodity for the economy as a whole equals the horizontally summed up marginal cost functions, whereas in the long run for each quantity supplied by the respective industry the supply price equals the minimum of the average cost function of the single firm.

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10 Fabio Petri remarked to us that the given number of firms in a General Equilibrium analysis starts with Hicks *Value and Capital*, where the temporary equilibrium framework made it quite reasonable. But in an intertemporal model there is no reason why the entry of firms in periods subsequent to the first one could not be accomodated, since to form a firm takes time.

Commodities are not only specified in terms of their physical characteristics, but also in terms of their date and location of availability. By means of this generalization of the concept of commodity Debreu sought to accommodate time and space into the model and thus construct a *general* theory: 'By focusing attention on changes of dates one obtains, *as a particular case* of the general theory of commodities ... a theory of saving, investment, capital, and interest. Similarly by focusing attention on changes of locations one obtains, *as another particular case* of the same general theory, a theory of location, transportation, international trade and exchange' (ibid., p. 32). Debreu assumed that there is only a finite number of distinguishable commodities (ibid.), which implies that the time-horizon of the model is finite. In a note appended to chapter 2 he admitted that 'there are, however, conceptual difficulties in postulating a predetermined instant beyond which all economic activity either ceases or is outside the scope of the analysis' (ibid., pp. 35-6; see also Malinvaud, 1953). In addition, it is assumed that 'the interval of time over which economic activity takes place is divided into a finite number of compact *elementary intervals* of equal length' (ibid., p. 29).

Debreu assumed that there exist current markets for *all* commodities, whatever their physical, temporal (within the given time horizon), or spatial specification.[11] Hence, in the 'economy' contemplated all trade for the entire time-horizon takes place at the beginning of the first period. If markets were reopened at later dates, then no additional trade would take place. As Arrow and Hahn stressed, the hypothesis that there exists a complete set of markets for current goods '"telescopes" the future into the present' (Arrow and Hahn, 1971, p. 33). Given a set of prices, each agent chooses a plan for all the elementary periods. An equilibrium for a 'private ownership economy' requires that all individual plans are, from the initial date onwards, mutually consistent for all future dates and compatible with 'the capital of the economy at the present instant', that is, initial endowments. Since Debreu assumes free disposal (1959, p. 42), in equilibrium some prices may be zero (in some periods): this concerns goods for which there is a negative excess demand.

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11 In the final chapter of his book, Debreu tried to cope also with the problem of uncertainty by generalizing the notion of commodity still further: a contract for the transfer of a commodity now includes in addition the specification of 'an event on the occurrence of which the transfer is conditional'. Debreu added: 'This new definition of a commodity allows one to obtain a theory of uncertainty free from any probability concept and formally identical with the theory of certainty developed in the preceding chapters' (ibid., p. 98). The theory makes use of Arrow's concept of 'choices of Nature'. In what follows we shall set aside this aspect.

Debreu's model exhibits several features that are disquieting. Here we cannot enter into a detailed discussion of these (see, however, Geanakoplos, 1987, Malinvaud, 1987, McKenzie, 1987, and Currie and Steedman, 1990, ch. 7). Some critical remarks must suffice. A major difficulty concerns the treatment of time. 'The principal objection to the restriction to a finite number of goods is that it requires a finite horizon and there is no natural way to choose the final period. Moreover, since there will be terminal stocks in the final period there is no natural way to value them without contemplating future periods in which they will be used' (McKenzie, 1987, p. 507). What Debreu in fact assumed in his formal model is that all economic activity stops at the arbitrarily given terminal instant, that is, resources existing at the end of the time-horizon have *zero* value. Because of the recursive structure of the model, all economic activities decided in the initial instant are derived with regard to the final period.

As regards the instant from which the economy is analyzed, that is, the 'present instant', the question arises whether there has been no economic activity prior to that date. Debreu's answer was in the negative: the economy is not created 'now'; it is rather assumed that, for the purpose of analyzing the economy's future development, the legacy of the past is exclusively *and* completely reflected in the amounts of resources inherited and the distribution of private ownership of these resources. In particular, it is assumed that there are no commitments carrying over from the past that constrain agents' present decisions. This implies of course that the logic of the model does not extend to the past, because otherwise Debreu would have to admit that at some dates in the past agents entered into contracts referring to dates that are still in the future. In addition there is the following conceptual problem pointed out by Joan Robinson and others. If in equilibrium some of the capital stocks turn out to be in excess supply these stocks assume zero prices. This possibility appears to indicate that the expectations entrepreneurs held in the past when deciding to build up the present capital stocks are not realized. Hence, strictly speaking we are faced with a *dis*equilibrium situation because otherwise the wrong stocks could not have been accumulated. Therefore, the problem arises how the past or, more exactly, possible discrepancies between expectations and facts influence the future.

As we have seen, earlier neoclassical authors, most notably Walras, were concerned with the long- and short-run equilibrium relationships between the prices of durable capital goods and the prices of their services, that is, the rates of return on different kinds of capital goods, and whether the short-run relationships gravitate towards some long-run relationship characterized by a uniform rate of return throughout the economy. To this effect Walras proposed an explicit *tâtonnement* procedure which he conjectured converged to long-period equilibrium. These concerns are not present in the Debreu model. It is not even asked how the economy is supposed ever to get into equilibrium. The notion of equilibrium is simply one of simultaneous clearing of all markets; there is no discussion of any *adjustment process* when defining equilibrium. Hence, in the Debreu analysis, as opposed to that presented by Walras with its long-period orientation, general equilibrium cannot be thought of as a 'centre of gravitation'.

*Infinite time horizon*

Until a few decades ago the time horizon in intertemporal general equilibrium theory was assumed to be finite and, therefore, arbitrary.[12] The introduction of an *infinite* horizon turned out to be critical (see also Burgstaller, 1994, pp. 43-8). It pushed the analysis towards steady-state analysis. (It ought to be stressed that the latter is a special case of long-period analysis and must not be identified with it.) This was clearly spelled out, for instance, by Robert Lucas in a contribution to the theories of endogenous growth. Lucas observed that 'for *any* initial capital $K(0) > 0$, the optimal capital-consumption path $(K(t), c(t))$ will converge to the balanced path asymptotically. That is, the balanced path will be a good approximation to any actual path "most" of the time' and that 'this is exactly the reason why the balanced path is interesting to us' (Lucas, 1988, p. 11). Lucas thus advocated a *(re-)switching* from an intertemporal analysis to a steady-state one. Since the balanced path of the intertemporal model is the only path analyzed by Lucas, the intertemporal model may be regarded simply as a step to obtain a rigorous long-period (steady-state) setting. (Paraphrasing a dictum put forward by Paul Samuelson in a different context, we may say that intertemporal analysis is a *detour* with regard to steady-state analysis).

Moreover, Lucas abandoned one of the characteristic features of all neoclassical theories, that is, income distribution is determined by demand and supply of factors of production: if we concentrate on the 'balanced path', capital in the initial period *cannot* be taken as given along with other 'initial endowments'. Since distribution cannot be determined by demand and supply of capital and labour, in Lucas's model it is determined in the following way. Labour is just the vehicle of 'human capital', that is, a producible factor, hence all factors are producible and the rate of profit is determined as in Chapter II of Sraffa's Book. This is not surprising since the assumption of a given real wage rate is formally equivalent to the assumption that there is a technology producing 'labour'. The 'human capital' story could be seen as just a rethoric artifice to render the idea of a given real wage more palatable to modern scholars. As regards its basic analytical structure (as opposed to its building blocks), some of the so-called 'new' growth theories belong within the realm of what we have called 'classical' economics. In particular, in the free competition versions of the theory the 'technology' to produce 'human capital' (or,

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12 The study of intertemporal models with an infinite time horizon was begun by Bewley (1972).

alternatively, 'knowledge' in some approaches) plays the same role as the assumption of a given real wage rate in 'classical' economics.

This leads to two questions. First, in the case in which we are exclusively, or at least mainly, interested in the steady state of an intertemporal model, and this steady state can be directly analysed using the long-period method, is it convenient to embark on a full-fletched intertemporal analysis? Or it is more convenient to study directly the long-period position? Note that if the answer to the last question is 'yes', we cannot but use the classical idea of an asymmetrical determination of distribution.

Second, are there issues that can easily be treated using the long-period method, while it would be difficult to treat them in an intertemporal analysis? With regard to those cases where this question has to be answered in the positive, there are a number of economic phenomena which can be grasped by using the long-period method, whereas they would remain obscure (at least temporarily) if this method were not used. This idea has been expressed in the literature. For instance, Burmeister (1996, p. 1346) has argued:

> It is natural to try to answer the easiest questions first, and it is much easier to study economies in a "long-period equilibrium" than ones in which the rate of profit is not uniform and is changing over time.

As is well known, the proof of the pudding is in the eating. Hence two simple examples will suffice to answer the above questions with regard to the special case of steady-states.

## 6. The usefulness of the long-period method: two examples

We begin with an example which deals with a multisector version of the AK model known from the literature on endogenous growth. In line with Lucas's argument referred to at the end of Section 5 we may ask: Why do we have to make the bold assumptions required to study an intertemporal model if, in the end, we are only interested in the steady state of that model? Why not use the long-period method directly? This does not mean, of course, that the problem whether and how an economic system gravitates towards a long-period position or, in the present case, a steady-state growth path is unintersting. Not at all. However, this problem is very difficult and will be set aside here. It suffices to remark that in order to study it we would need a truly dynamic analysis in which agents can make errors etc. It hardly needs to be stressed that the intertemporal equilibrium model does not constitute such a dynamic analysis.

*Endogenous growth: a multisector version of the AK model*

Let us consider an economy with the conventional representative agent who is faced with the following problem ($t \in$ R):

$$\max \int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma} - 1}{1 - \sigma} \mathrm{d}t \qquad (1a)$$

$$\text{s. to } \mathbf{x}_t^T(\mathbf{I} - \delta\mathbf{A}) \geqq C_t\mathbf{e}_1^T + \dot{\mathbf{x}}_t^T\mathbf{A}, \qquad (1b)$$

$$\mathbf{x}_\mathrm{t} \geqq \mathbf{0}, \mathbf{x}_0^T\mathbf{A} \leqq \bar{\mathbf{x}}, C_t \geq 0, \qquad (1c)$$

where $\rho > 0$ is rate of time preference, $1/\sigma$ is the elasticity of substitution between present and future consumption ($1 \neq \sigma > 0$), $C_t$ is the consumption of commodity 1, the only consumption good, at time $t$, $\mathbf{A}$ is an $n$x$n$ instantaneous capital goods matrix, the corresponding $n$x$n$ instantaneous output matrix being $\mathbf{I} - (1 - \delta)\mathbf{A}$, where $\mathbf{I}$ is the $n$x$n$ identity matrix and $\delta$ is the uniform rate of depreciation of capital goods, $0 \leq \delta < 1$ (that is, no primary factor is used in production and there is no choice of technique[13]), $\mathbf{x}_\mathrm{t}$ is the vector of intensities of operation of processes defined by matrix $\mathbf{A}$ and depreciation rate $\delta$, $\dot{\mathbf{x}}$ is the derivative of $\mathbf{x}$ with respect to time, $\bar{\mathbf{x}}$ is the given positive vector of initial stocks of commodities, and $\mathbf{e}_1$ is the first unit vector. Matrix $\mathbf{A}$ is assumed to be nonnegative and indecomposable. The $\rho$ is assumed to satisfy the inequalities

$$(\lambda^{-1} - \delta)(1 - \sigma) < \rho \leq (\lambda^{-1} - \delta) \qquad (2)$$

13 In a framework assuming discrete time it would be possible to assume $\delta = 1$. In such a framework this would mean that all capital is consumed in one unit of time, that is, all capital is circulating capital. This would not be possible in a continuous time framework, because in it $\delta = 1$ would mean that capital is consumed at the same instant of time at which produced commodities appear: with no time elapsing between inputs and outputs, there would actually be no capital! Moreover, in order to allow for a situation in which a capital good is consumed in a finite amount of time we would need to introduce an infinite number of commodities for each type of capital good, each of this infinite number of commodities representing the capital good at the appropriate (continuous) vintage. With continuous time, then, the idea that a capital good depreciates in the sense that a part of it evaporates is not only the simplest one available to capture the idea of capital, but also the only one which, as far as we know, avoids the need to have recourse to an infinite number of capital goods.

where $\lambda$ is the eigenvalue of maximum modulus (also known as the Frobenius eigenvalue, cf. Takayama, 1974, ch. 4; or Kurz and Salvadori, 1995, pp. 509-19) of matrix **A**. It will also be assumed for simplicity that **A** is invertible and has $n$ distinct eigenvalues. By using optimal control theory it is possible to show

**Proposition 1.** There are scalars $g > 0$ and $C_0 \geq 0$ such that $\hat{\mathbf{x}} = \mathbf{x}_0 e^{gt}$ and $\hat{C} = C_0 e^{gt}$ are solutions to problem (1) if and only if

$$g = \frac{1 - \delta\lambda - \rho\lambda}{\lambda\sigma} \tag{3}$$

and there is a scalar $\theta > 0$ such that

$$\bar{\mathbf{x}} = \theta \mathbf{e}_1^T[\mathbf{I} - (\delta + g)\mathbf{A}]^{-1}\mathbf{A}.$$

For a proof, see Salvadori (1998).[14]

Let us now consider another economy with the same technology. As in the previous exercise it is assumed that only commodity 1 is consumed and that the saving-investment mechanism determines the following relationship between the real profit rate $r$ and the growth rate $g$

$$g = \frac{r - \rho}{\sigma}, \tag{4}$$

whatever the meaning of $\rho$ and $\sigma$. Note that these assumptions are much less strong than those of the previous exercise. If some agent owns the commodities $\mathbf{e}_j^T\mathbf{A}$ at time 0 and uses them to produce continuously commodity $j$ from time 0 to time $t$, so that (i) at time $t$ he owns the commodities $e^{-\delta t}\mathbf{e}_j^T\mathbf{A}$; and (ii) at each time $\tau$, $0 \leq \tau < t$, he has a flow of product of $e^{-\delta\tau}$ units of commodity $j$ which is invested in another business; and if all investments earn a nominal rate of profit $i$ (to be determined); then

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14 Freni, Gozzi and Salvadori (1999) have investigated more deeply a variant of this model in order to analyse the dynamics outside the steady-state solution. They found also that if the second of the inequalities (2) is not satisfied some other long-period position can be found with a negative growth rate: the only process which is operated in the interest of consumption is process 1 and the growth rate is equal to or lower than $-\delta$. We refrain here from showing how these steady-state solutions are found using the long-period method.

$$\int_0^t e^{(t-\tau)i} e^{-\delta\tau} \mathbf{e}_j^T \mathbf{p}_\tau d\tau + e^{-\delta t} \mathbf{e}_j^T \mathbf{A}\mathbf{p}_t = e^{it} \mathbf{e}_j^T \mathbf{A}\mathbf{p}_0. \tag{5}$$

In a long-period position relative prices are constant and the rate of inflation is also constant, so that for each $t$

$$\mathbf{p}_t = e^{\pi t}\mathbf{p},$$

where $\mathbf{p}$ is a vector to be determined and $\pi$ is the rate of inflation (or deflation). Hence, if long-period conditions are assumed to hold (and if $i \neq \pi - \delta$),[15] from equation (5) we obtain

$$[e^{it} - e^{(\pi-\delta)t}] \left[ \frac{1}{i - \pi + \delta} \mathbf{e}_j^T \mathbf{p} - \mathbf{e}_j^T \mathbf{A}\mathbf{p} \right] = 0,$$

which can be written as

$$[e^{(r+\pi)t} - e^{(\pi-\delta)t}] \left[ \frac{1}{r + \delta} \mathbf{e}_j^T \mathbf{p} - \mathbf{e}_j^T \mathbf{A}\mathbf{p} \right] = 0,$$

where $r$ is the real rate of profit ($r = i - \pi$). Since this equation must hold for each $j$ we have

$$\left[ \frac{1}{r + \delta} \mathbf{p} - \mathbf{A}\mathbf{p} \right] = \mathbf{0}.$$

Since $\mathbf{p}$ must be semipositive, then we have from the Perron-Frobenius Theorem that $\mathbf{p} > \mathbf{0}$ is the right eigenvector of matrix $\mathbf{A}$ corresponding to the eigenvalue of maximum modulus $\lambda > 0$, and

$$r = \frac{1 - \delta\lambda}{\lambda} > 0,$$

the inequality being a consequence of the second of inequalities (2). Moreover, because of equation (4), equation (3) holds. Note that inequalities (2) imply

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15 If $i = \pi - \delta$, then from equation (5) we would obtain

$$te^{it}\mathbf{e}_j^T\mathbf{p} = 0,$$

which can hold for each $t$ only if $\mathbf{e}_j^T\mathbf{p} = 0$. And since this equation should hold for each $j$, $\mathbf{p}$ could not be semipositive.

$$0 \leq g < r.$$

Finally, since only commodity 1 is consumed and the economy grows at rate $g$, the consumption $C_t$ and the intensity vector $\mathbf{x}_t$ must satisfy the following equations

$$C_t = C_0 e^{gt},$$

$$\mathbf{x}_t^T = C_0 \mathbf{e}_1^T [\mathbf{I} - (\delta + g)\mathbf{A}]^{-1} e^{gt},$$

provided that $\delta + g < \lambda^{-1}$, which is certainly the case, since the first of inequalities (2) holds.

This completes the demonstration that one can analyse the steady-state properties of a multisector variant of the AK-model without being compelled to invoke the bold assumptions necessary in an intertemporal model.

We turn now to our second example. This concerns the case of jointly utilized fixed capital items. (For a more detailed discussion, see Kurz and Salvadori, 1995, chapter 9.)

*Jointly utilized fixed capital items*

It is assumed that there are $m$ processes and $n$ commodities. Each process of production $i$ ($i$ = 1, 2, . . ., $m$) is defined by the triplet ($\mathbf{a}_i$, $\mathbf{b}_i$, $l_i$), where $\mathbf{a}_i = (a_{i1}, a_{i2}, \ldots, a_{in})^T$ is the nonnegative material input vector, $\mathbf{b}_i = (b_{i1}, b_{i2}, \ldots, b_{in})^T$ is the nonnegative output vector, and $l_i$, a scalar, is the nonnegative labour input. Thus, the whole technology is defined by the triplet ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{l}$), where

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{a}_m^T \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{b}_1^T \\ \mathbf{b}_2^T \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{b}_m^T \end{bmatrix}, \quad \mathbf{l} = \begin{bmatrix} l_1 \\ l_2 \\ \cdot \\ \cdot \\ \cdot \\ l_m \end{bmatrix}.$$

It is assumed that

*Assumption 1*: It is not possible to produce something without using some material inputs, i.e.,

$$\mathbf{e}_j^T \mathbf{A} \geq \mathbf{0}. \qquad j = 1, 2, \ldots, m$$

*Assumption 2*: All commodities are producible, i.e.,

$$\mathbf{B}\mathbf{e}_j \geq \mathbf{0}. \qquad j = 1, 2, \ldots, n$$

*Assumption 3*: Labour is indispensable for the reproduction of commodities, i.e.,

$$(\mathbf{x} \geq \mathbf{0},\ \mathbf{x}^T(\mathbf{B} - \mathbf{A}) \geqq \mathbf{0}) \Rightarrow \mathbf{x}^T\mathbf{l} > 0.$$

The example we are going to analyze concerns jointly utilized fixed capital items that are subject to the assumption that they cannot be transfered among sectors, that is, an oven once utilized to produce bread cannot be used during its lifetime to produce biscuits.[16] On this assumption a number of "desirable" properties will be shown to hold. Among these properties there is the fact that consumption patterns do not matter at all in determining prices or operated processes, whereas the growth rate can play a role in determining the cost minimizing technique and, therefore, prices. The assumption we are going to investigate is the following.

*Assumption 4*: There are two subsets, S and T, of the set of commodities N such that

(A.4.1) $S \cap T = \emptyset$, $S \cup T = N$;

(A.4.2) commodities in T are never consumed;

(A.4.3) each process produces one and only one commodity in S;

(A.4.4) if commodity $i \in$ T is produced by process $j$ producing commodity $h \in$ S, then there is no process producing a commodity $k \in$ S, $k \neq h$, such that it either produces $i$ or utilizes $i$ as an input;

(A.4.5) for each process producing commodity $j \in$ T there is a process with the same inputs and the same outputs except that commodity $j$ is not produced.

It is immediately recognized that if and only if single production holds, Assumption 4 is satisfied with S = N and T = ∅. On the contrary, if Assumption 4 holds with S ≠ N and T ≠ ∅, then, for the sake of simplicity, we can refer to the commodities in T as 'old machines' and to the commodities in S as 'finished goods'. The rationale for the above axioms can now be stated as follows. Axiom (A.4.1) implies that a commodity is either an old machine or a finished good, but never both. Axiom (A.4.2) implies that old machines are never consumed. Axiom (A.4.3) rules out joint production proper. Axiom (A.4.4) states that old machines cannot be transferred among sectors. Axiom (A.4.5) implies that old machines can be disposed of at no cost (leaving

16 A case in which machines are transferable among sectors, but are not used jointly, has been recently analyzed by Salvadori (1999).

no scrap behind), that is there is free disposal of old machines, but not necessarily of finished goods.

If Assumption 4 holds, we may reorder commodities and processes in the following way: the first $s$ commodities are in S, the next $t_1$ commodities are in T and are produced jointly with commodity 1, the next $t_2$ commodities are in T and are produced jointly with commodity 2, . . . , the next $t_s$ commodities are in T and are produced jointly with commodity $s$, some $t_i$ may be equal to zero, $t_1 + t_2 + \ldots + t_s = t$, $s + t = n$; the first $m_1$ processes produce commodity 1, the next $m_2$ processes produce commodity 2, . . . , the next $m_s$ processes produce commodity $s$, $m_1 + m_2 + \ldots + m_s = \text{m}$. In order to simplify the notation, let us also introduce $t_0 = 0$ and $m_0 = 0$.

Therefore Axiom A.4.4, i.e. old machines cannot be transferred among sectors, is equivalent to the following properties of matrices **A** and **B**:

(i) if $k \in$ S, $b_{hk} > 0$ if and only if $\sum_{j=0}^{k-1} m_j < h \leq \sum_{j=0}^{k} m_j$;

(ii) if $k \in$ S and $s + \sum_{j=0}^{k-1} t_j < i \leq s + \sum_{j=0}^{k} t_j$, then $b_{hi} > 0$ or $a_{hi} > 0$ only if $\sum_{j=0}^{k-1} m_j < h \leq \sum_{j=0}^{k} m_j$.

In Figures 1 and 2 matrices **A** and **B**, respectively, are represented on the assumption that $s = 5$ and that all commodities except commodity 3 are produced by using old machines: grey areas represent nonnegative elements, white areas represent zero elements, and black areas represent positive elements.

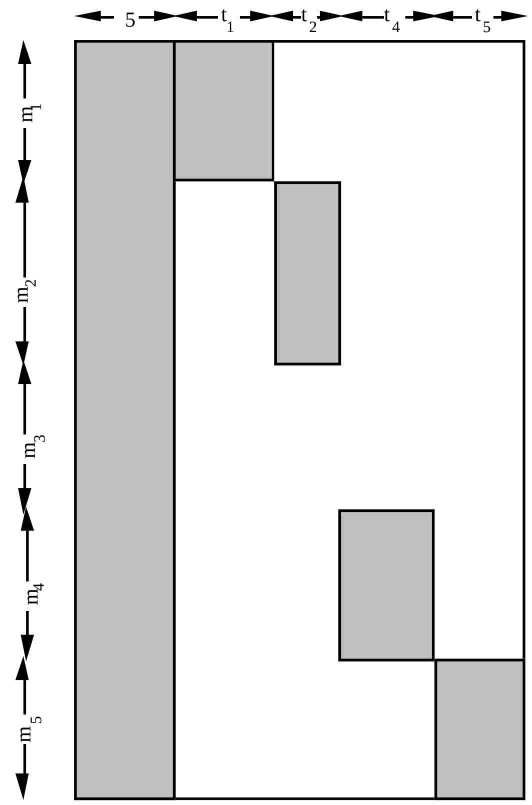


Figure 1

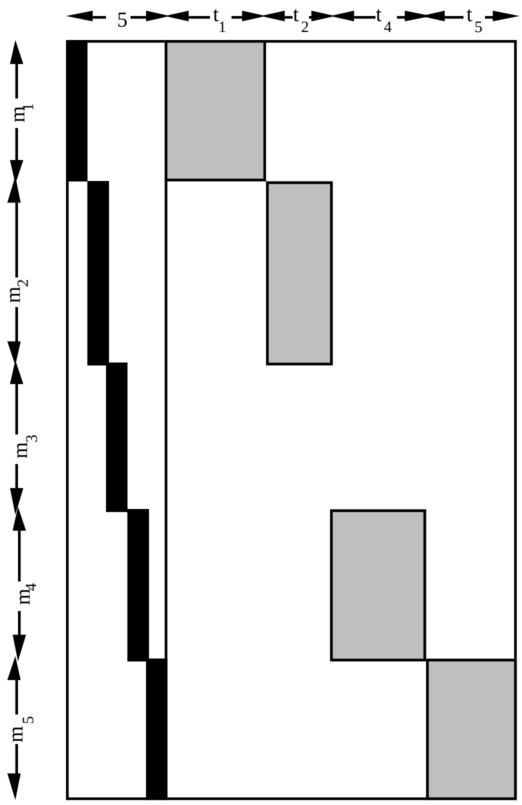


Figure 2

We say that there is a long period position corresponding to rate of profit r and demand function **d**(r, **p**, **x**) if there is a solution to the following set of equations and inequalities.

$$[\mathbf{B} - (1 + r)\mathbf{A}]\mathbf{p} \leqq \mathbf{l} \qquad (6a)$$

$$\mathbf{x}^T[\mathbf{B} - \mathbf{A}] \geqq \mathbf{d}(r, \mathbf{p}, \mathbf{x})^T \qquad (6b)$$

$$\mathbf{x}^T[\mathbf{B} - (1 + r)\mathbf{A}]\mathbf{p} = \mathbf{x}^T\mathbf{l} \qquad (6c)$$

$$\mathbf{x}^T[\mathbf{B} - \mathbf{A}]\mathbf{p} = \mathbf{d}(r, \mathbf{p}, \mathbf{x})^T\mathbf{p} \qquad (6d)$$

$$\mathbf{x} \geqq \mathbf{0}, \ \ \mathbf{p} \geqq \mathbf{0}. \qquad (6e)$$

Since we want to prove that prices are independent from $\mathbf{d}(r, \mathbf{p}, \mathbf{x})$ provided the economy is growing at a uniform rate which will be called $g$ and that Assumption 4 holds, it will be assumed that $\mathbf{d}(r, \mathbf{p}, \mathbf{x}) = g\mathbf{x}^T\mathbf{A} + \mathbf{c}(r, \mathbf{p}, \mathbf{x})^T$, where the last $t$ elements of vector $\mathbf{c}(r, \mathbf{p}, \mathbf{x})$ are identically nought. Hence system (6) is better stated as

$$[\mathbf{B} - (1 + r)\mathbf{A}]\mathbf{p} \leqq \mathbf{l} \qquad (7a)$$

$$\mathbf{x}^T[\mathbf{B} - (1 + g)\mathbf{A}] \geqq \mathbf{c}(r, \mathbf{p}, \mathbf{x})^T \qquad (7b)$$

$$\mathbf{x}^T[\mathbf{B} - (1 + r)\mathbf{A}]\mathbf{p} = \mathbf{x}^T\mathbf{l} \tag{7c}$$

$$\mathbf{x}^T[\mathbf{B} - (1 + g)\mathbf{A}]\mathbf{p} = \mathbf{c}(r, \mathbf{p}, \mathbf{x})^T\mathbf{p} \tag{7d}$$

$$\mathbf{x} \geqq \mathbf{0}, \;\; \mathbf{p} \geqq \mathbf{0}. \tag{7e}$$

Furthermore function $\mathbf{c}(r, \mathbf{p}, \mathbf{x})$ satisfies the following obvious assumptions

*Assumption 5*: Function $\mathbf{c}(r, \mathbf{p}, \mathbf{x})$

(A.5.1) is continuous in $\mathbf{x}$;

(A.5.2) is homogeneous of degree 1 in $\mathbf{x}$, that is, it satisfies the equality

$$\alpha\mathbf{c}(r, \mathbf{p}, \mathbf{x}) = \mathbf{c}(r, \mathbf{p}, \alpha\mathbf{x})$$

for each $\alpha \geq 0$, $r \geq 0$, $\mathbf{p} \geq \mathbf{0}$, $\mathbf{x} \geq \mathbf{0}$;

(A.5.3) is nonnegative everywhere it is defined, that is,

$$\mathbf{c}(r, \mathbf{p}, \mathbf{x}) \geqq \mathbf{0}$$

for each $r \geq 0$, $\mathbf{p} \geq \mathbf{0}$, $\mathbf{x} \geq \mathbf{0}$;

(A.5.4) satisfies Walras's law, that is

$$\mathbf{c}(r, \mathbf{p}, \mathbf{x})^T\mathbf{p} = \mathbf{x}^T\mathbf{l} + (r - g)\mathbf{x}^T\mathbf{A}\mathbf{p}$$

for each $r \geq 0$, $\mathbf{p} \geq \mathbf{0}$, $\mathbf{x} \geq \mathbf{0}$.

Axioms (A.5.1), (A.5.2), and (A.5.4) are needed in order to allow the existence of a uniform growth rate which is constant over time, whereas Axiom (A.5.3) just asserts that a negative amount of a commodity cannot be consumed. Before stating conditions for the existence and uniqueness of a cost-minimizing technique some Lemmata will be introduced.

*Lemma 5*: If the following Assumption 6 holds, and if $g \leq r$, then the following system of equations and inequalities has a solution:

$$[\mathbf{B} - (1 + r)\mathbf{A}]\mathbf{y} \leqq \mathbf{l} \tag{8a}$$

$$\mathbf{q}^T[\mathbf{B} - (1 + g)\mathbf{A}] \geqq \mathbf{a}^T \tag{8b}$$

$$\mathbf{q}^\mathrm{T}[\mathbf{B} - (1 + r)\mathbf{A}]\mathbf{y} = \mathbf{q}^T\mathbf{l} \tag{8c}$$

$$\mathbf{q}^T[\mathbf{B} - (1 + g)\mathbf{A}]\mathbf{y} = \mathbf{a}^T\mathbf{y} \tag{8d}$$

$$\mathbf{q} \geqq \mathbf{0},\ \ \mathbf{y} \geqq \mathbf{0}, \tag{8e}$$

where $\mathbf{a}$ is a given semipositive vector.

*Proof*: See Lippi (1979) or Salvadori (1980).

*Assumption 6*: There is a vector $\mathbf{z}$ such that

$$\mathbf{z} \geqq \mathbf{0}, \quad \mathbf{z}^T[\mathbf{B} - (1 + r)\mathbf{A}] \geqq \mathbf{a}^T.$$

The following two Lemmata introduce the procedure which will be followed to derive the consequences of Assumption 4.

*Lemma 7*: Let Assumptions 4 and 6 hold, let $g \leq r$, and let ($\mathbf{q}^*$, $\mathbf{y}^*$) be a solution to system (8) for $\mathbf{a} = \hat{\mathbf{a}}$, where $\hat{\mathbf{a}}$ is a vector with each of the first $s$ elements being equal to 1, and each of the others being equal to 0. Then there is a vector $\mathbf{q}^{**}$ such that ($\mathbf{q}^{**}$, $\mathbf{y}^*$) is a solution to system (8) for $\mathbf{a} = (\mathbf{c}_s^T, \mathbf{0}^T)$, where $\mathbf{c}_s$ is any semipositive vector in $R^s$.

*Proof*: Let $\mathbf{Q} = [q_{hk}]$ be an $s$x$m$ matrix such that

$$q_{hk} = \begin{cases} \mathbf{e}_h^T\mathbf{q}^* & \text{if } \sum_{i=0}^{h-1} m_i < k \leq \sum_{i=0}^{h} m_i \\ 0 & \text{elsewhere} \end{cases}$$

Let us define the matrices $\mathbf{D}_0$, $\mathbf{C}_0$, $\mathbf{D}_1$, $\mathbf{C}_1$, obtained by the following partition of matrices $\mathbf{QA}$ and $\mathbf{QB}$: ($\mathbf{D}_0$, $\mathbf{D}_1$) = $\mathbf{QB}$, ($\mathbf{C}_0$, $\mathbf{C}_1$) = $\mathbf{QA}$, $\mathbf{C}_0$ and $\mathbf{D}_0$ being square. Since Assumption 4 holds and since the first s entries of $\hat{\mathbf{a}}$ are equal to 1, all the others being equal to zero,

$$\mathbf{D}_0, \mathbf{C}_0, \mathbf{D}_1, \mathbf{C}_1 \text{ are nonnegative, } \mathbf{D}_0 \text{ is diagonal, diag } \mathbf{D}_0 > \mathbf{0};$$

$$[\mathbf{D}_0 - (1 + r)\mathbf{C}_0]\mathbf{y}_s^* + [\mathbf{D}_1 - (1 + r)\mathbf{C}_1]\mathbf{y}_t^* = \mathbf{Ql}; \tag{9}$$

$$[\mathbf{D}_1 - (1 + g)\mathbf{C}_1] \geqq \mathbf{0},\ \ [\mathbf{D}_1 - (1 + g)\mathbf{C}_1]\mathbf{y}_t^* = \mathbf{0}; \tag{10}$$

$$\mathbf{e}^T[\mathbf{D}_0 - (1 + g)\mathbf{C}_0] \geqq \mathbf{e}^T; \tag{11}$$

where $(\mathbf{y}_s^{*T}, \mathbf{y}_t^{*T}) = \mathbf{y}^{*T}$. It is immediately recognized, because of (10), that if there is a nonnegative solution $\mathbf{v}^*$ to the equation

$$\mathbf{v}^{\mathrm{T}}[\mathbf{D}_0 - (1 + g)\mathbf{C}_0] = \mathbf{c}_s^T,$$

then vector $\mathbf{q}^{**} = \mathbf{Q}^T\mathbf{v}^*$ satisfies the Lemma. To prove that $\mathbf{v}^*$ exists, it is enough to remark that matrix $\mathbf{C}_0\mathbf{D}_0^{-1}$ is nonnegative and that inequality

$$\mathbf{e}^T[\mathbf{I} - (1 + g)\mathbf{C}_0\mathbf{D}_0^{-1}] > \mathbf{0}^T$$

holds, because of (11); then we obtain from a well known theorem that matrix $[\mathbf{D}_0 - (1 + g)\mathbf{C}_0]$ is invertible and

$$[\mathbf{D}_0 - (1 + g)\mathbf{C}_0]^{-1} \geq \mathbf{0}. \qquad (12)$$

Q. E. D.

*Lemma 8*: Let Assumptions 4 and 6 hold and let $\mathbf{y}^* = (\mathbf{y}_s^{*T}, \mathbf{y}_t^{*T})^T$ be defined as in Lemma 7, then $\mathbf{y}_s^{*T} > \mathbf{0}$ and the weak inequality (11) is satisfied as an equation.

*Proof*: Let $(\mathbf{q}_i^{**}, \mathbf{y}^*)$ be a solution to system (8) for $\mathbf{a} = \mathbf{e}_i$, $i \in \{1, 2, ..., s\}$. Thus,

$$\mathbf{q}_i^{**T}[\mathbf{B} - (1 + g)\mathbf{A}]\mathbf{y}^* = \mathbf{e}_i^T\,\mathbf{y}^*$$

$$\mathbf{q}_i^{**T}[\mathbf{B} - (1 + r)\mathbf{A}]\mathbf{y}^* = \mathbf{q}_i^{**T}\mathbf{l}$$

i.e.,

$$\mathbf{e}_i^T\mathbf{y}^* = \mathbf{q}_i^{**T}\mathbf{l} + (r - g)\,\mathbf{q}_i^{**T}\mathbf{A}\mathbf{y}^*.$$

Then, the first part of the Lemma is obtained since $(r - g) \geq 0$ and $\mathbf{q}_i^{**T}\mathbf{l} > 0$ because of Assumption 3. The second part of the Lemma is an immediate consequence of the first part.

Q. E. D.

Now the main theorem concerning the existence and irrelevance of the form of function $\mathbf{c}(r, \mathbf{p}, \mathbf{x})$, apart from the elements mentioned in Assumption 5 and in Axiom (A.4.2), can be proved.

*Theorem 9*: If Assumptions 4 and 5 hold and $g \leq r$, then Assumption 6 is sufficient for the existence of a long-period position corresponding to rate of profit $r$ and demand function $g\mathbf{x}^T\mathbf{A}\mathbf{p} + \mathbf{c}(r, \mathbf{p}, \mathbf{x})$. The operated processes and the price vector $\mathbf{p}$ in the long-period position are independent of function $\mathbf{c}(r, \mathbf{p}, \mathbf{x})$.

*Proof*: Axiom (A.4.2) implies that

$$\mathbf{c}(r, \mathbf{p}, \mathbf{x}) = \begin{bmatrix} \mathbf{c}_s(r, \mathbf{p}, \mathbf{x}) \\ \mathbf{0} \end{bmatrix}$$

where sub-vector $\mathbf{c}_s(r, \mathbf{p}, \mathbf{x})$ has size s and is a function of $r$, $\mathbf{p}$, $\mathbf{x}$, which is continuous and homogeneous of degree 1 in $\mathbf{x}$ (because of Axioms (A.5.1) and (A.5.2)), nonnegative for nonnegative values of $r$, $\mathbf{p}$, $\mathbf{x}$ (because of Axiom (A.5.3)), and such that

$$\mathbf{c}_s^{\mathrm{T}}(r, \mathbf{p}, \mathbf{x})\mathbf{p}_s = \mathbf{x}^T\mathbf{l} + (r - g)\mathbf{x}^T\mathbf{A}\mathbf{p} \tag{13}$$

where $\mathbf{p}_s$ is the vector consisting of the first $s$ elements of vector $\mathbf{p}$ (because of Axiom (A.5.4)). Then, because of Lemma 7 it is enough to prove that there is a semipositive vector $\mathbf{v}$ such that

$$\mathbf{v}^T[\mathbf{D}_0 - (1 + g)\mathbf{C}_0] = \mathbf{c}_s^{\mathrm{T}}(r, \mathbf{y}^*, \mathbf{Q}^T\mathbf{v}),$$

that is that function

$$\mathbf{v}(\mathbf{u})^T := \mathbf{c}_s^{\mathrm{T}}(r, \mathbf{y}^*, \mathbf{Q}^T\mathbf{u})[\mathbf{D}_0 - (1 + g)\mathbf{C}_0]^{-1}$$

has a fixed point. In order to prove this, let us consider the set

$$\mathrm{S} = \{\mathbf{u} \in \mathrm{R}^s \mid \mathbf{u} \geqq \mathbf{0}, \mathbf{u}^T[\mathbf{D}_0 - (1 + g)\mathbf{C}_0]\mathbf{y}_s^* = 1\}.$$

It is immediately checked that if $\mathbf{u}$ is in S, then $\mathbf{v}(\mathbf{u})$ is also in S, and since it is continuous, it has in S a fixed point. In fact, if $\mathbf{u}$ is in S, then $\mathbf{v}(\mathbf{u}) \geqq \mathbf{0}$ since inequality (12) and Axiom (A.5.3) hold and

$$\mathbf{v}(\mathbf{u})^T[\mathbf{D}_0 - (1 + g)\mathbf{C}_0]\mathbf{y}_s^* = \mathbf{c}_s^{\mathrm{T}}(r, \mathbf{y}^*, \mathbf{Q}^T\mathbf{u})\mathbf{y}_s^* = \mathbf{u}^T\mathbf{Q}\mathbf{l} + (r - g)\mathbf{u}^T\mathbf{Q}\mathbf{A}\mathbf{y}^* =$$

$$= \mathbf{u}^T[\mathbf{D}_0 - (1 + g)\mathbf{C}_0]\mathbf{y}_s^* = 1$$

since equations (13) (second equality) and (9) and (10) (third equality) hold. Q. E. D.

This shows that given the rate of profits and the uniform rate of growth, we can determine which technique will be chosen by cost-minimizing producers and the system of relative prices associated with that technique even in the presence of sophisticated (and realistic) assumptions about fixed capital. In the conditions specified, consumption patterns have no impact on the processes operated and prices.

## 7. Conclusion

The paper has two parts. In the first part (Sections 2-5) we provide a short summary account of the developments in the theory of value and distribution that took place since the beginnings of systematic economic analysis at the time of the classical economists. We distinguish between a 'classical' approach to the theory of value and distribution and a 'neoclassical' one. The emphasis is on the distinguishing features of the two approaches. It is argued that the classical authors from Smith to Ricardo developed what is known as the method of 'long-period positions' of the economic system, focusing attention on the persistent and systematically operating factors shaping the economy at any given moment of time and over time. This method was adopted by basically all neoclassical authors up until the late 1920s and has made a reappearance in more recent times especially with the so-called 'new' growth theories. It had been abandoned essentially because of internal problems of the theory originating from its concept of a 'quantity of capital' as a magnitude that could be given independently of relative prices and income distribution. In terms of the methods of intertemporal equilibrium and temporary equilibrium, championed by Firedrich August von Hayek, Erik Lindahl and John Richard Hicks, it was sought to overcome the capital theoretic difficulties and yet preserve the demand and supply approach to the problem of income distribution. It is argued that these alternatives are beset by a number of disquieting features.

The second part (Section 6) is designed to show the power of the traditional long-period method in terms of two examples. The first example concerns a multi-sector variant of the 'linear' or 'AK growth model'; it is shown that its steady-state properties can be analysed in relatively simple terms, whereas in an intertemporal setting some very bold assumptions are required to do so. The second case refers to the case of fixed capital within a classical framework of the analysis, in which several fixed capital items are jointly utilized.

We hope to have made clear (i) that the long-period method is an extremely powerful tool of analysis, if handled correctly; and (ii) that a correct long-period analysis cannot take the endowment of 'capital' as given. However, our tribute to long-period analysis of 'classical' derivation must not be mistaken to imply an opposition on our part to the development of a proper dynamical analysis. We are rather convinced that a correct long-period analysis provides the best ground for starting to elaborate a dynamical analysis. As Edwin Burmeister recently stressed: 'Very little is known about the properties of such more realistic economies ..., and even the little that is known usually is only about special and quite unrealistic cases (such as the one-good case). Almost nothing is known about the dynamic behavior of the more complex models' (Burmeister, 1996, p. 1346) which can be studied within a long-period classical framework.

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