

# Fund-flow versus flow-flow in production theory

## Reflections on Georgescu-Roegen's Contribution

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### **Abstract**

The paper discusses Nicholas Georgescu-Roegen's contribution to production theory, comparing it with the contributions of John von Neumann and Piero Sraffa and emphasizing the problem of the choice of technique. It is shown that Georgescu-Roegen's fund-flow approach, in which fixed capital is taken to be an "immutable agent," can be misleading and cannot address issues that are easily handled by the alternative flow-flow approach that reduces fixed capital to circulating within a joint-products framework and allows adequate treatment of the wear and tear of machines as well as the problem of capital utilization.

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The reasonable man adapts himself to the world; the unreasonable one persists in trying to adapt the world to himself. Therefore all progress depends on the unreasonable man.

(G. B. Shaw)

### **1. Introduction**

Nicholas Georgescu-Roegen (1970, p. 9) ended his 1969 Richard T. Ely Lecture on "The Economics of Production" with the observation:

The economics of production, its elementary nature notwithstanding, is not a domain, [sic] where one runs no risk of committing some respectable errors. In

fact, the history of every science, including economics, teaches us that the elementary is the hotbed of the errors that count most.

In Georgescu-Roegen's view neoclassical production theory was just such a "hotbed." He was therefore keen to elaborate an alternative to it. Another author with a critical stance toward neoclassical theory was Piero Sraffa (1960), who had likewise developed an alternative conceptualization starting from the physical real cost approach of the classical economists.<sup>1</sup> Therefore it is perhaps interesting to see how the contributions of the two authors compare with regard to the critical and the constructive task they set themselves. In this paper the emphasis will be predominantly on Georgescu-Roegen's approach to the theory of production. We shall examine his works and check his more analytical propositions against the background of Sraffa's analysis. For convenience, we shall take Georgescu-Roegen's 1969 Ely Lecture as a useful point of reference whenever this is possible.<sup>2</sup>

The structure of the paper is as follows; in Section 2 we shall explore the concept of the production function and the criticism levelled at it. Section 3 turns to Georgescu-Roegen's alternative approach to the theory of production, comparing it with the approach of the classical economists that was revived by John von Neumann and Piero Sraffa. Section 4 counterposes Georgescu-Roegen's distinction between funds and flows, on the one hand, and the distinction between fixed and circulating capital goods, on the other. Section 5 compares the fund-flow approach with what may be called the "flow-flow" approach, in which fixed capital is reduced to circulating capital within a joint-products framework. Section 6 establishes the superiority of the flow-flow approach in terms of a discussion of the important problem of the choice of technique; it is shown that the fund-flow approach may fail to identify the cost-minimizing technique. Section 7 contains some concluding remarks.

## 2. The production function

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<sup>1</sup> Before Sraffa, John von Neumann ([1937] 1945) had put forward in print an approach that shares salient features with the one elaborated by the classical economists [see Kurz and Salvadori (1993)]. As we know from his unpublished papers kept at Trinity College Library, Cambridge (U.K.), Sraffa had developed his first systems of equations of production as early as the second half of 1927 [see Kurz (1998)]. Sraffa also assisted David Champernowne when the latter composed his commentary on the English translation of von Neumann's paper for the *Review of Economic Studies*.

<sup>2</sup> For a more detailed exposition of his ideas on production, see Georgescu-Roegen (1971, chap. IX). See also Dos Santos Ferreira (1974).

In his Ely lecture and elsewhere, Georgescu-Roegen criticized the "analytical imbroglio" of the conventional production function [see also Georgescu-Roegen (1971, pp. 241-4)].<sup>3</sup> He objected that the time factor is often ignored, that stocks and rates of flow tend to get confounded,<sup>4</sup> and that production functions are constructions of economic theorists and not "data" taken from engineers or industrial chemists. This criticism prompted him to some epistemological and methodological considerations. He expressed the view that any analytical representation of a production process is bound to be partial. To require such a representation to be complete in the sense that each and every aspect pertinent to a production process is taken into consideration would amount to taking into account the "seamless absolute whole" – an impossible task. He stressed that "analysis must proceed by some heroic simplifications and totally ignore their ultimate consequences" (1970, p. 2). This does not mean that anything goes. It means that, since not everything can be taken into account, there must be a judicious selection of aspects to be dealt with, which he notes requires "some intimate knowledge of the corresponding phenomenal domain" (p. 3). The theorist cannot avoid specifying the "analytical boundary" of the process of production: without such a boundary, there can be no analysis. The former is the "basic element" of the latter. In this way he claims that "some slits [are] cut into the absolute whole" (p. 2).

The conventional concept of the production function was criticized in the so-called Cambridge controversies in the theory of capital.<sup>5</sup> Contrary to a widespread misconception, the criticism concerned not only Solow-type aggregate production functions, but *all* attempts to represent amounts of heterogeneous capital goods by quantities of a single factor "capital" that is independent of income distribution and relative prices. This concept of capital comes essentially in two versions. The first, contending that a scalar representation of vectors of heterogeneous means of production can be found, had as its most prominent advocate Eugen von Böhm-Bawerk with his concept of the "average period of production." According to the second version the amount of capital employed in the economy as a whole can be conceived of as a *value* magnitude. Variants of this version were put forward, for example, by John Bates Clark

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<sup>3</sup> According to Georgescu-Roegen (1970, p. 1), the starting point is Wicksteed ([1894] 1992). It should be mentioned, however, that a linear homogeneous production function is present already in Wicksell (1893); see Sandelin (1976).

<sup>4</sup> A remarkable exception is Winston (1982).

<sup>5</sup> See, for example, Kurz and Salvadori (1995, ch. 14; 1998).

and Alfred Marshall. The capital critique of the 1960s and 1970s established clearly that neither version can be sustained.<sup>6</sup>

Surprisingly, Georgescu-Roegen made hardly any attempt to relate his criticism to the one put forward in the Cambridge controversies. The statements that come perhaps closest to mentioning implicitly the latter start from his objection to Marx's concept of "congealed abstract capital." He added, "As a highly abstract simile, the standard form of the Neoclassical production function – as a function of  $K$ , the cardinal measure of homogenous 'capital,' and  $H$ , the cardinal measure of homogenous 'labor' – is not completely useless." This statement could easily be misunderstood, but the one immediately following it produces clarity: "the value of the standard form of the production function as a blueprint of reality is nil. It is absurd therefore to hold on to it in practical applications" (1971, p. 244). The rock on which the neoclassical production function is said to founder is that heterogeneous capital goods "have no common measure."<sup>7</sup> Georgescu-Roegen concludes, "Marginal productivity ... comes out as an empty word" (ibid.).<sup>8</sup>

### 3. A different approach

In his 1970 paper Georgescu-Roegen argued that the analytical boundary consists of two components, one that separates the process from the rest of reality while the other specifies the duration of the process, because a timeless *process* is obviously a *contradictio in adjecto*. It comes as no surprise that he was opposed to the concept of instantaneous production as encountered, for example, in Walras. According to Georgescu-Roegen, the following aspects

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<sup>6</sup> See also Fisher (1993).

<sup>7</sup> This is not quite true: the common measure of goods are their prices. The real problem is that prices depend on income distribution. Therefore, no measure of heterogeneous capital can be found that is independent of the latter.

<sup>8</sup> In this context it is perhaps interesting to note that as early as in a note dated 19 June 1943 Sraffa pointed out that the marginalist authors are precluded from raising objections to the labour theory of value: "For the Marginal Product theory of capital *presupposes, implicitly, that Hypothesis*" (D3/12/34: 33, Sraffa's emphasis; see also D3/12/16: 34). Sraffa was thus clear at an early stage of his investigation that the "parable" of neoclassical theory presupposes the "realism" of the labor theory of value; see Samuelson's (1962) attempt to support the neoclassical theory in terms of the "surrogate production function." (The references are to Sraffa's unpublished papers and follow the catalogue prepared by Jonathan Smith, archivist. We should like to thank Pierangelo Garegnani, literary executor of Sraffa's papers, for granting us permission to quote from them.)

are crucial. First, "Because of the principle, 'No analytical boundary, no analytical process', analysis must renounce any hope of including in this description the happenings inside an analytical process" (1970, p. 3), once the boundaries of the *elementary* process have been specified. Second, he insisted that another "heroic step" must be taken, which consists of assuming that the number of elements involved is finite and that every element is cardinally measurable. He stressed, "Each element of an analytical process – as we have decided – must be completely homogeneous." This condition, he added, "does not always cover sameness" (p. 4). The reference is to durable instruments of production and workers, both of which are liable to undergo some change during the production process: tools are subject to wear and tear and workers invariably become tired.

In Georgescu-Roegen's view, these two cases "raise a troublesome problem for the economist." This is so because the economist's "commodity fetishism" (p. 4) presumes given qualities of the elements considered in the analysis and is thus incompatible with qualitative change. However, contrary to what the reader might expect, this is not followed by an attack on commodity fetishism. Georgescu-Roegen rather defends his vision of the "economist":

Our entire analytical edifice would collapse if we were to accept the alternative position that the aim of economic production is to produce not only the usual products but also tired workers and used tools. (Ibid.)

This view is difficult to sustain and, as we shall see, has serious implications for the approach elaborated by Georgescu-Roegen. Also, while some economists do perhaps fit his description, others certainly do not. More important, it is an undoubtable *fact* (which is quite independent from whether it could also be an "aim") that production over any length of time generates tired workers and partly worn-out tools. And facts, it seems, ought to be taken as what they are. In one place Georgescu-Roegen stressed that his concern was "mainly with the problem of valid analytical representation of the *relations among facts*" (1992, p. 130, emphasis added; see also 1971, pp. 217-18). Unfortunately, his fund-flow approach does not follow this maxim (see Section 4).

Among the economists who naturally allowed for used tools are John von Neumann and Piero Sraffa with their treatment of fixed capital in a joint-products framework. Accordingly, the production process is subdivided in elementary processes, with a main product exiting from production at the end of each.<sup>9</sup> In addition, there may be a by-product: a new fixed capital good

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<sup>9</sup> In the case of pure joint production the output consists of several products.

such as a machine that enters the production process is considered as a particular input that, in principle, is not different from circulating capital goods, such as raw materials, while the older machine available at the end of an elementary process is considered as a particular joint output of the process; the older machine then enters as an input in one of the other elementary processes. The young and the old machine are thus treated as different commodities, which involves reducing fixed to circulating capital. This way to deal with fixed capital is able to account for the fact that tools are subject to wear and tear. Moreover, if, as Georgescu-Roegen often emphasized, a machine enters and exits from production, then the subdivision in elementary processes needs to be more detailed: The exit of not only a main product but also a machine from production or the entry of any machine (old or new) into production must signal the end (and the beginning) of an elementary process. In this case semi-finished products need to be defined as hypothetical commodities since all machines and all semi-finished products exit the elementary process (to enter into other elementary processes), and even if they are not sold and bought, they need to be priced.

Sraffa did not deal with tired workers in the same way as with used machines, but there is no obvious reason why dealing with it could not be done. Perhaps von Neumann had it in mind when constructing his model.<sup>10</sup> In this case production processes may be considered as producing, among other things, tired workers, whereas separate "recreation processes" could describe a part of the activity within households, with tired workers entering as inputs (jointly with commodity inputs and services) and "fresh workers" exiting as outputs. In the sense used in the literature on the Leontief model,<sup>11</sup> a model with these features would be more closed (less open) than the usual model without such recreation processes and thus without tired workers as outputs of the usual production processes.<sup>12</sup> In other words, one may formulate systems of production showing both how the "factors" produce the commodities and the commodities (plus other inputs) produce the factors (including labor power).

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<sup>10</sup> "Consumption of goods takes place only through the process of production which includes necessities of life consumed by workers and employees" (von Neumann, 1945, p. 2).

<sup>11</sup> In the "closed" Leontief model the household sector is treated as if it were an ordinary industry. Thermodynamically, the system is, of course, not closed, because it relies on geothermal and solar energy. In Leontief's economic accounting these are set aside; that is, they are treated as free goods. Therefore, in this respect the "closed Leontief model" is open!

<sup>12</sup> There are other important aspects which will be set aside here; one of them concerns learning by doing and training on the job. This latter aspect is related to the problem of the formation of "human capital," a topic beyond the scope of this paper; see, however, Kurz and Salvadori (1995, ch. 11).

Strangely, Georgescu-Roegen, who otherwise advocated a strictly objectivist-materialist point of view, in this regard chose an "idealistic" way out of what he considered to be an impasse. He (1970, p. 4) contended,

A new heroic step is needed to eliminate this difficulty. It consists of the familiar, old fiction of a process in which capital is maintained constant. The fiction does raise some analytical issues, for if all tools and all workers are to be maintained at a constant level of efficiency, any production process will have to include most of the enterprises and households in the world.

What Georgescu-Roegen presumably had in mind when referring to the "familiar, old fiction" is a vertically fully integrated industry which produces a net amount of a particular commodity while also reproducing all the means of production directly and indirectly used up in the production of that net amount. Actually, his concept of vertical integration would have to be still more comprehensive because it necessitates also the reproduction of the physical and mental state of the workers when they entered the production process. Since what Georgescu-Roegen calls "fund" is but another name for his concept of keeping capital or any other factor of production fully intact, the latter deserves careful scrutiny.

Were Georgescu-Roegen to imply that "maintaining capital constant" involves keeping each and every durable means of production at its "level of efficiency" when brand-new, then the following objections arise: 1) it is doubtful that this requirement is at all technically feasible; and if it were, 2) it would make no sense economically, because in most (all?) cases it would be a wasteful exercise. Georgescu-Roegen appears to have been aware that such an extreme specification of the concept cannot sensibly be sustained. However, was this enough to sound the retreat? For this is what Georgescu-Roegen did when he wrote: "A simple glance at the activity inside a plant or a household suffices to convince us that efforts are constantly directed not toward keeping durable goods physically self-identical (which is quite impossible [sic]), but toward maintaining them in good working condition. And this is all that counts in production" (p. 4).

This specification is rather vague and therefore not very useful. Something more can be said. If we adopt the joint-products approach to fixed capital, the concept of "maintaining capital constant" can be given a clear analytical expression. In fact, it is possible to combine the elementary processes producing a final good (for the sake of simplicity, we set aside the problem of joint production of final goods) in such proportions that the quantities of used machines entering the whole set of processes is exactly equal to the quantities exiting the whole set of processes. It is important to stress that the proportions mentioned depend on the rate of

growth of the economic system and change with that rate. In this way the capital stock is maintained not because each machine is maintained at its original efficiency, but because the physical changes of machines are taken into account in such a way that the set of machines is the same even if each single machine changes its role within the system as a consequence of its wear and tear and some old machines are disposed of and some new machines are introduced.<sup>13</sup>

#### 4. Funds and flows vs. fixed and circulating capital

The above considerations led Georgescu-Roegen to the distinction between *funds* and *flows*. A fund is characterized by "economic invariableness", because it enters a production process and (ideally) comes out "without any impairment of its economic efficiency" (1970, p. 5): it is taken to be perfectly "maintained" during the process. Flows on the other hand appear only as inputs or as outputs. Examples of funds are "Ricardian land" possessed of "original and indestructible powers" ( $L$ ), instruments and tools of production, or "capital equipment" (ibid., p. 5) ( $K$ ), and workers or rather their human capital alias labor power ( $H$ ). Input flows include raw materials ( $R$ ), circulating capital goods ( $I$ ) and inflows "earmarked for maintenance" ( $M$ ); output flows include products ( $Q$ ) and waste ( $W$ ). Georgescu-Roegen stressed that the production process so defined is a "reproducible process"; and that its description is incomplete if it does not contain both categories – funds, or "immutable agents," and flows.

The upshot of his argument is the well-known analytical representation of a reproducible process

$$[R_0^T(t), I_0^T(t), M_0^T(t), Q_0^T(t), W_0^T(t); L_0^T(t), K_0^T(t), H_0^T(t)]$$

as a point in abstract functional space. The time element is specified: the process has a beginning and an end, and its duration is finite. This description is said to be "a far cry from the notion inherited from Wicksteed, according to which a process is represented by a point in the ordinary (Euclidian) space" (p. 5).

The theorist, initially concerned with providing a complete representation of a process, cannot stop there but "has to decide which elements may be left out because they are economically

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<sup>13</sup> This concept is fully compatible with the Smithian notion of the division of labour, which, as Georgescu-Roegen rightly stressed, heralds the factory system, "one of the greatest *economic* inventions in the history of mankind" (1971, p. 248). See also Georgescu-Roegen (1965, section IV; 1971, ch. IX, sections 9 and 11).

irrelevant" (ibid.). This selection implies that the description of a process will normally not comply with the laws of physics. Georgescu-Roegen noted this fact in terms of two examples: natural resources that are available in abundance and are thus free goods, and waste products that are subject to free disposal.<sup>14</sup>

The underlying premises make perfect sense when discussing levels of production that are "small" compared with the amounts of the best qualities of natural resources available, on the one hand, and on the other levels of waste generation that are "small" compared with the environment into which the waste is dumped. Even in conditions where these premises no longer hold, there may still be good reasons for setting aside the problems of the scarcity of natural resources and of the costliness of waste disposal when discussing certain problems. This does not mean that one has necessarily fallen victim to the twin fallacy of regarding nature as both a horn of plenty and a bottomless sink. Georgescu-Roegen's warnings that one must not lose sight of the laws of thermodynamics are justified. But a clarification is perhaps useful with regard to his statement that "waste by definition has no value" (ibid.). Obviously, a waste product can only be a joint output of some other output. Moreover, the waste of some process may be a useful input in some other process(es) (e.g., manure): "Where there's muck, there's brass!" The important point is that in general it is impossible to say that a product is "waste" (or "bad") on the basis of a partial view of the economic system. The character of a joint product – whether it has a positive price or not – is decided only for the system of production and consumption as a whole. Without taking into consideration the overall quantities required (or effectually demanded) of the different products and the methods of production available from which cost-minimizing producers can choose, including the methods to recycle precious materials, this question cannot be answered. If certain amounts of a product cannot be used in consumption or production, then the superfluous quantity of it needs to be eliminated. Such disposal processes are generally not free, but costly. In this case the waste product under consideration will fetch a *negative* price, because in order to get rid of the product, labor and

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<sup>14</sup> See also the following statement: "It is the idea of capital equipment *being kept as a constant fund by the very process in which it participates*. Strictly interpreted, this idea is a fiction. A process by which something would remain indefinitely outside the influence of the Entropy Law is factually absurd. But the merits of the fiction are beyond question" (Georgescu-Roegen, 1971, p. 229; emphasis in the original). However, somewhat later in the same volume Georgescu-Roegen was very critical of the assumption of production as a circular flow as it was advocated by the Physiocrats, the classical economists and Marx. He contended that "no other conception could be further from the correct interpretation of facts" (ibid., p. 281). Here we cannot enter into a detailed discussion of this problem. It must suffice to remark that the usefulness of an assumption is reflected in the findings one is able to derive with its help. For a treatment of the problem of exhaustible resources in terms of the classical approach to the theory of production, see Kurz and Salvadori (2000, 2001).

other kinds of inputs have to be employed. Therefore, waste products will often have a negative value; only in exceptional cases can they be treated as free goods.

Let us now turn briefly to Georgescu-Roegen's distinction between factors *I* and *K*. Both consist of produced means of production, or capital goods, many of which will be reproduced. They differ in terms of their longevity: while the former exhaust their powers during the (elementary) production period, which is set by the theorist, the latter survive more than one period and, according to the assumption of perfect maintenance, are indeed conceived of as an everlasting source of productive services. The fund-flow distinction is here equivalent to that between perennial and circulating capital.

At least since the time of the Physiocrats, economists have been aware of the fact that fixed capital introduces a formidable complication into the theory of value: while the circulating part of capital advances, contributing entirely to the annual output, in effect "disappears" from the scene, the contribution of the durable part is less obvious and can only be imputed in correspondence to its wear and tear. This imputation problem appears to be also at the bottom of Georgescu-Roegen's fund-flow distinction as applied to produced means of production: "While it is true that the cloth – the inflow element – effectively passes into the coat, the same cannot apply to the needle – a fund element. ... The point is that the problem of how the contribution of a fund affects the value of the product is not as directly simple as in the case of a flow factor" (1970, p. 5). This cannot be disputed. Yet, again, more can be said.

## **5. The "fund-flow" and the "flow-flow" approach to fixed capital**

While Georgescu-Roegen did not treat fixed capital in terms of the joint-products method, he was certainly familiar with von Neumann's paper on equi-proportionate growth. In fact, he referred to it [see, for example, Georgescu-Roegen {1966, p. 311, n. 16}]. However, he provided no reason for not conceiving, for example, of a  $t$  year old tractor as a joint output of corn produced by means of a  $t - 1$  year old tractor. This method allows one to determine the price of each machine at each point in time (where time is taken as a discrete variable) and therefore the time profile of the price (or rather book value) of the ageing tractor. Implicit in this is the depreciation pattern of the machine. This method does not presuppose the singularly special case of constant efficiency throughout the tractor's economic life. Moreover, the approach is not restricted to the case of a single fixed capital item utilized in a process, but can also cope with the empirically important case of jointly utilized durable instruments of production (see Kurz and Salvadori, 1995, chs 7 and 9). Nevertheless, in the literature one encounters the view that Georgescu-Roegen's approach to the theory of production is superior

to alternative approaches, including the one adopted by von Neumann and Sraffa [see, for example, Maneschi and Zamagni (1997, p. 702, especially n. 7) and Piacentini (1995, pp. 466-7)]. In the following we will critically scrutinize this view.

The von Neumann-Sraffa approach to fixed capital consists in formalizing the old classical idea of treating old machines left at the end of each period as economically different goods from the machines which entered production at the beginning of the period. This approach to the problem of fixed capital may be dubbed "flow-flow". It avoids the bold assumption of the "fund-flow" approach that funds leave the process just as they had entered it.

In the flow-flow approach with joint products, the problem of the choice of technique is dealt with in the following way. In standard notation, the price vector  $\mathbf{p}$  and the intensity vector  $\mathbf{x}$  will be determined jointly with the wage rate  $w$  by the system

$$[\mathbf{B} - (1 + r)\mathbf{A}]\mathbf{p} \leq w\mathbf{l} \quad (1)$$

$$\mathbf{x}^T[\mathbf{B} - (1 + r)\mathbf{A}]\mathbf{p} = w\mathbf{x}^T\mathbf{l} \quad (2)$$

$$\mathbf{x}^T[\mathbf{B} - (1 + g)\mathbf{A}] \geq \mathbf{d}^T \quad (3)$$

$$\mathbf{x}^T[\mathbf{B} - (1 + g)\mathbf{A}]\mathbf{p} = \mathbf{d}^T\mathbf{p} \quad (4)$$

$$\mathbf{p} \geq \mathbf{0}, \mathbf{x} \geq \mathbf{0}, w \geq 0, \mathbf{q}^T\mathbf{p} = 1 \quad (5)$$

where  $\mathbf{A}$  is the material input matrix,  $\mathbf{l}$  is the labor input vector,  $\mathbf{B}$  is the output matrix,  $\mathbf{d} \geq \mathbf{0}$  is the consumption vector,  $r$  is the rate of interest, and  $g$  is the uniform rate of growth. Inequality (1) implies that no process is able to pay extra profits. Equation (2) implies, also because of inequalities (5), that if a process is not able to pay the given rate of interest, it is not operated. Inequality (3) implies that the total demand (consumption and gross investment) is satisfied.<sup>15</sup> Equation (4), also because of inequalities (5), is the Rule of Free Goods: overproduced commodities fetch a zero price. Equation (5) fixes the numéraire, where  $\mathbf{q}$  is any given semipositive vector.

The special case in which there is no joint production of finished goods, but there is, of course, the joint production of one finished good and one or several specific old machines, holds if and

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<sup>15</sup> If  $\mathbf{x}$  is the vector of the intensities of operation of the different processes at time  $t$ , then  $(1 + g)\mathbf{x}$  is the vector of the intensities of operation at time  $t + 1$ .

only if an assumption is made about the conformation of matrices **A** and **B**.<sup>16</sup> The interested reader can find the whole story in Kurz and Salvadori (1995, chs. 7 and 9). Here it must suffice to illustrate the issue in terms of an example. If there are five finished goods, four of which (the first, the second, the fourth and the fifth) are produced using fixed capital, for each finished good  $i$ ,  $m_i$  processes to produce it and  $t_i$  used machines specific to it are available ( $i = 1, 2, \dots, 5$ ). The matrices **A** and **B** are conformed as in figures 1 and 2, respectively, where grey areas represent nonnegative elements, white areas zero elements, and black areas positive elements.

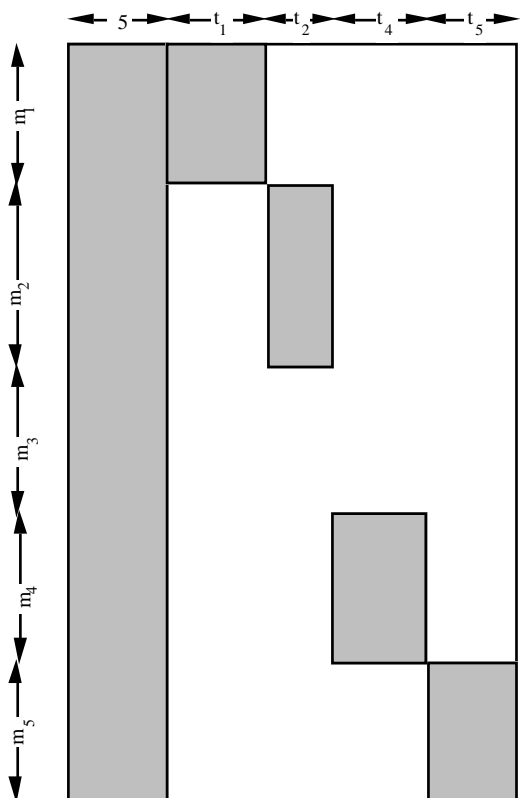


Figure 1: Matrix A

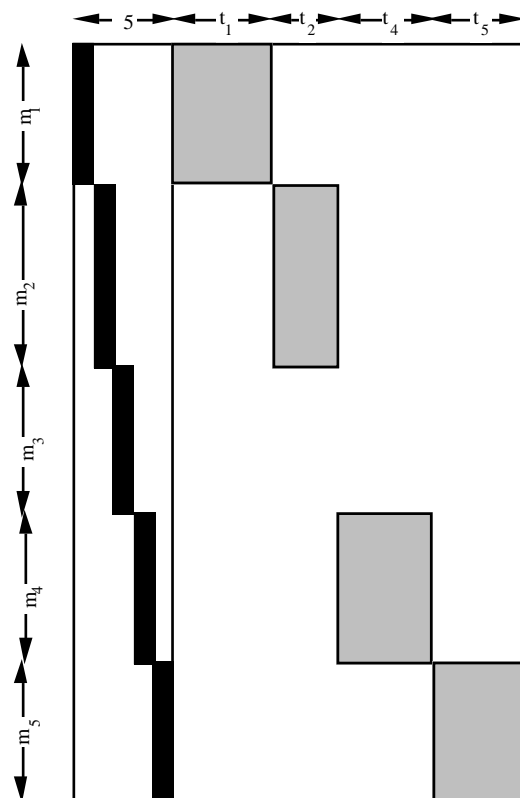


Figure 2: Matrix B

This is enough to distinguish commodities in "finished goods" and "old machines" and to prove a number of good properties including the one that prices do not depend on the vector of consumed commodities, whereas they may depend on the growth rate when machines are jointly utilized. However, a proper analytical treatment of "depreciation" requires some additional assumptions. It is in fact necessary to connect all used machines and some "finished

<sup>16</sup> The assumption that old machines are specific is generally called "intransferability of old machines" in this literature. Salvadori (1999) proves that, at least in the case in which machines are not jointly used, this assumption can be replaced by the more reasonable one that the efficiency of non-specific (or transferable) old machines does not depend on the sector in which they are used.

goods," interpreted as "new machines," among themselves in order to identify them as vintages of some "types of machine." [See Kurz and Salvadori (1995, pp. 260-61)]

The treatment of fixed capital in a joint-products framework, of course, does not do away with Georgescu-Roegen's important distinction between the "*agents* of a process" of production (funds) and its flow elements "which are *used* or *acted upon* by the agents" (1971, p. 230; emphasis in the original). Hence there is no presumption that by analytically reducing fixed capital to circulating, the former becomes substitutable against the elements of circulating capital as conventionally defined. Fund and flow elements are essentially complementary, as Georgescu-Roegen rightly stressed. The corresponding distinction between "finished goods" and "used machines" emphasizes the fact that only some finished goods may be interpreted as material to be transformed, whereas both "new machines" and "used machines" cannot be so.

In this section we restrict ourselves to stressing that there are several problems concerning fixed capital which cannot be investigated in terms of a formalism in which fixed capital does not wear out. There is both the choice of the *economic lifetime* of a fixed capital item and the choice of its *pattern of utilization*. The possibility of a machine becoming economically obsolete before the end of its technically feasible lifetime (which may be taken to be infinite) is obviously ruled out in cases of constant or increasing efficiency (setting aside technical progress, of course). With decreasing or changing efficiency, however, premature truncation is advantageous as soon as the price (or book value) of a partly worn out instrument of production becomes negative. Since the price of a machine (whether new or "aged") is equal to the capital value one gets by discounting all future net receipts that may be obtained by further use of it, where the actual rate of interest (profit) is taken as the discount rate, negative prices would indicate "losses". The optimal truncation date of a fixed capital item generally depends on income distribution (the rate of interest). The utilization aspect exhibits both an extensive and an intensive dimension. The former relates to the number of units within a given time period (day, week) during which a durable capital good is actually operated – for example, whether a single- or a double-shift scheme is adopted; the latter relates to the intensity of operation per unit of active time (hour) of the item, for example, the speed at which a machine is run.<sup>17</sup> The economic lifetime of a fixed capital good and the pattern of its operation are, of course, not independent of one another.

Let us now see what Georgescu-Roegen had to say about the problem of the utilization of funds. We read, for example:

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<sup>17</sup> Georgescu-Roegen was fully aware of these aspects (see, especially, Georgescu-Roegen, 1971, pp. 246-7).

I contend, one of the most important aspects of the economics of production is how to minimize these periods of fund idleness, whether we are thinking of man, capital equipment, or land. (1970, p. 6; see also 1971, pp. 247-8)

He added that "the economics of production reduces to two commandments: first, produce by the factory system [i.e., by arrangement in parallel] and, second, let the factory operate around the clock" (1970, p. 8). These are intriguing statements and Georgescu-Roegen himself seems to have felt the need to qualify them. He pointed out that due to seasonal variations, in large parts of husbandry the factory system cannot be adopted. Hence, what can at most be done is to try "to render the idleness of the agents as small as possible in each particular case" (p. 9). He went on to maintain:

The second commandment is particularly relevant for the underdeveloped economies. In a rich country, it makes perfect sense to operate every factory with one shift, even if the shift be of six or four hours only. In a rich country, there is also no need for night shifts, except whenever technology imposes around-the-clock production. Briefly, in a rich country leisure is a commodity which people may prefer to higher income. (Ibid.)

The length of the working day is said to be "the simplest and the most direct lever of economic development" (ibid.).

While we do not deny that raising the degree of capital utilization may be a good strategy to accelerate the process of developing an economy, we have reservations with regard to some of Georgescu-Roegen's other propositions. First, in deciding the degree of capital utilization, several factors besides those already mentioned matter. The view that in competitive conditions there is a tendency to utilize expensive fixed capital as much as possible can be traced back far in the history of our subject. John Stuart Mill ([1848] 1965, p. 131) went so far as to argue that keeping machines working for twenty-four hours "is evidently the only economical mode of employing them."<sup>18</sup> However, at any given moment of time there may be cultural, institutional, political and economic factors at work that prevent the utilization of capital around the clock. For example, input flows per unit of output and input and output prices are generally not time-invariant; many of these quantities and prices tend to change during the day, month or year. Take the simple case of shift work, where the ordinary wage per hour plus a shift premium has to be paid to workers during the night shift despite the fact that the productivity of labor at night

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<sup>18</sup> The same view was advocated by socialist authors concerned with the problem of their economies catching up in the debate about rapid industrialisation.

is often smaller than during the day due to the disturbance of man's biorhythm. Such a wage premium and the lower productivity are potential deterrents to adopting a double- or treble-shift system.<sup>19</sup> We are therefore skeptical about the validity of Georgescu-Roegen's contention that if the proper utilization of capital "were realized by miracle overnight, we would discover that we have long since been planning for a world with an immense excess capacity of industrial production" (1970, p. 9). Perhaps, in normal conditions at least, competitive economic systems are not very far away from cost-minimizing degrees of capital utilization.<sup>20</sup>

Secondly, these factors may explain why in industrialised countries plant and equipment are left idle for much of the time quite independently of any potential deficiency of aggregate effective demand. Obsolete machines are no longer used because technical innovations followed by changes in relative prices make it no longer profitable to do so. People's preference for leisure over income, to which Georgescu-Roegen referred, has little to do with the less than full utilization of capital. It is true that leisure may be preferred over real income. Yet this need not be in conflict with an interest in generating a given real income in a cost-minimizing way. It is the behavior of firms, concerned with reducing costs of production, that in specific institutional and economic conditions accounts for a substantial part of observable idleness of plant and equipment.<sup>21</sup> If Georgescu-Roegen was right, then one would expect that the richer a country (in terms of per capita income), the smaller is its (average) degree of utilization of plant and equipment. However, empirically the opposite is true: a growing income per capita and a gradual reduction in average working hours per labourer are accompanied by a persistent upward trend in the overall degree of capital utilization [see, for example, Betancourt and Clague (1981) and Foss (1981)].

In this section we have summarized the flow-flow approach to fixed capital. We have shown that with its help certain economic problems can be analysed in a more satisfactory way than by using the fund-flow approach. Since we could not identify any aspect which can be tackled using the latter, but not the former, this is enough to decide in favor of the flow-flow approach.

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<sup>19</sup> For a discussion of the problem of capital utilization in a "classical" framework, see Kurz (1990, ch. 5) and Kurz and Salvadori (1995, ch. 7, section 7).

<sup>20</sup> This view would have to be qualified for economies that are characterised by a high degree of concentration, because oligopolies voluntarily tend to keep spare margins of capital utilization in order to deter potential competitors from entering the market.

<sup>21</sup> For the reasons given, there is no presumption that the (overall) degree of capital utilization desired by firms equals full utilization. The difference between the latter and the former may be considered "voluntary". Degrees of utilization that fall short of the desired one may (entirely or partly) be traced back to a lack of effective demand.

However, the difficulties besetting the conventional fund-flow approach are perhaps not yet sufficiently clear. We shall, therefore, in the next section deal with the important problem of the choice of technique in terms of a simple numerical example in order to show that even in cases in which *prima facie* the fund-flow approach seems to be equivalent to the flow-flow approach, this need not be the case: the fund-flow approach may be misleading in regard to the problem of identifying the cost-minimizing technique.

## 6. The problem of the choice of technique: a numerical example

Let us consider a technology which can be described using the flow-flow scheme of Table 1. Commodity  $C$  is produced by using two machines ( $M$  and  $N$ ) which last three and two years, respectively. (For the sake of simplicity it is assumed that the new  $M$  is indistinguishable from the new  $N$ , whereas the two durable instruments of production differ when they are old.) Processes (2)-(7) represent all possible arrangements. In process (2) the two new machines are used jointly, and two machines  $M$  and  $N$ , each one year old, are produced jointly with commodity  $C$ . In processes (4), (5), and (6) a new machine is used jointly with one old machine (a two year old  $M$ , a one year old  $N$ , a one year old  $M$ , respectively): the corresponding older machines are found among the outputs (a one year old  $N$ , a one year old  $M$ , a two year old  $M$  and a one year old  $N$ , respectively). In processes (3) and (7) only old machines are used; in particular, the one year old  $N$  is used jointly with a one or two year old  $M$ : the corresponding older machine  $M$  is found among the outputs of process (3), whereas process (7) does not include any output of machines. (We set aside the problem of scrap, which is equivalent to invoking free disposal.) All processes use labor and commodity  $C$  as inputs and all processes (2)-(7) produce commodity  $C$ .

The same technology can be described using a fund- or stock-flow description. Two new machines plus one unit of labor and 5 units of commodity  $C$  enter the stage. At the end of the year (the beginning of the second year) 15 units of commodity  $C$  exit, and one unit of labor enters. At the end of the second year (beginning of the third year) 25 units of commodity  $C$  exit, and one new machine and one unit of labor enter. At the end of the third year (the beginning of the fourth year) 25 units of commodity  $C$  exit, and one new machine and one unit of labor enter. The same happens at the end of the fourth and the fifth year (the beginning of the fifth and the sixth year). At the end of the sixth year 20 units of commodity  $C$  exit.

**Table 1**

	Commodity inputs							Outputs				
	$C$	New machines	$M_1$	$M_2$	$N_1$	labor		$C$	New machines	$M_1$	$M_2$	$N_1$
(1)	5	0	0	0	0	1	→	0	1	0	0	0
(2)	5	2	0	0	0	1	→	20	0	1	0	1
(3)	5	0	1	0	1	1	→	30	0	0	1	0
(4)	5	1	0	1	0	1	→	30	0	0	0	1
(5)	5	1	0	0	1	1	→	30	0	1	0	0
(6)	5	1	1	0	0	1	→	30	0	0	1	1
(7)	5	0	0	1	1	1	→	20	0	0	0	0

The two descriptions seem to do the job equally well. But this is not so, as will be shown in a steady-state framework in which the rate of growth,  $g$ , is given from outside the system. The six processes of the flow-flow description are unified in one entity in the stock-flow description, hiding that this is actually not the cost-minimizing choice. In a cost-minimizing technique, if there are five commodities ( $C$ , the new machine(s),  $M_1$ ,  $M_2$ ,  $N_1$ ) only 5 processes are operated, unless the other two are linearly dependent processes [see Kurz and Salvadori (1995, p. 258)]. In fact, in the flow-flow description processes (2)-(7) do not need to be operated one after the other: they could be operated *side by side*, that is, in parallel. But when this is done, it is immediately observed that instead of operating processes (2) and (7) it is possible to rearrange the same inputs in order to operate again processes (4) and (5), thereby obtaining a larger output in terms of corn. The reader must not rush to the conclusion that only processes (1) and (3)-(6) can be operated. In fact, processes (3)-(6) cannot sustain large growth rates, and for high values of  $g$  process (2) is required in order to increase the production of one year old machines. When this is the case, process (3) cannot be operated, since it is the largest consumer of one year old machines. Mathematical proofs of these claims can be found in the appendix.

This shows that the fund-flow approach to the problem of fixed capital may be misleading as regards the problem of the choice of the cost-minimizing technique. The flow-flow approach adopted in this paper allows instead for a correct solution. It also deserves to be emphasized again that the problem of choice of technique cannot generally be answered without taking into

consideration the economic environment, in particular, whether the economy is growing and at what rate.

There are other important economic issues that can be properly treated only in terms of a satisfactory analysis of the problem of the choice of technique. Think, for example, of the case of idle land. We set aside the valid reasons given by Georgescu-Roegen (1971, pp. 250-3) for the idleness of workers and land inherent in agriculture due to the seasonal rhythm of each region on earth. As is well-known since the classical economists and especially David Ricardo's contribution to the theory of the rent of land, it may be perfectly sensible from an economic, cost-minimizing point of view not to employ certain qualities of land and instead cultivate some other qualities, some perhaps even intensively.<sup>22</sup> It is not fully clear which concept of "scarcity" Georgescu-Roegen held. Occasionally one gets the impression that he tended to speak of scarcity whenever a natural resource happens to be available in an absolutely limited amount only. The economic concept of scarcity is different from this physical notion. As Ricardo ([1817] 1951, p. 69) pointed out with regard to land available in abundant quantity: "no rent could be paid for such land, for the reason stated why nothing is given for the use of air and water, or for any of the gifts of nature which exists in boundless quantity." Hence, what matters from an economic point of view is how the available (known and finite) amount of a natural resource compares with the effectual demand for it.

## **7. Concluding remarks**

The paper discussed Georgescu-Roegen's approach to the theory of production against the background of the approach of the "classical" economists, which has been revived and generalised by John von Neumann and Piero Sraffa. The emphasis is on the treatment of fixed capital and the problem of the choice of technique. It is shown that Georgescu-Roegen's "fund-flow" approach runs into difficulties when facing the latter problem. This is illustrated in terms of a numerical example: the fund-flow approach may give the wrong answer to the problem of cost-minimizing choice of technique. Essentially the difficulty is that the fund-flow model, even when it appears to represent the same structure of production as the flow-flow model, makes illegitimate prior assumptions about which of the underlying techniques will be used. In the von Neumann-Sraffa approach fixed capital is dealt with in a joint-products framework, which amounts to reducing fixed to circulating capital. The "flow-flow" approach allows for a correct

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<sup>22</sup> For a treatment of the role of land in production and rent, see, for example, Kurz and Salvadori (1995, ch. 10).

solution of the choice of technique problem. In addition the paper comments on the problem of capital utilization.

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## Appendix

Let  $g^*$  be the unique positive real solution of the equation

$$g^3 + 3g^2 + 2g - 1 = 0,$$

i.e.

$$g^* = \sqrt[3]{\frac{1}{2} + \frac{1}{6} \sqrt{\frac{23}{3}}} + \sqrt[3]{\frac{1}{2} - \frac{1}{6} \sqrt{\frac{23}{3}}} - 1 \approx 0.32472.$$

Let  $G$  be the unique positive real solution of the equation ( $G \approx 0.80298$ )

$$27 + 14g - 25g^2 - 31g^3 - 13g^4 - 2g^5 = 0$$

and  $R_1$  be the unique positive real solution of the equation ( $R_1 \approx 0.859404$ )

$$25 + 18r - 19r^2 - 29r^3 - 13r^4 - 2r^5 = 0.$$

The reader can easily check that on the assumption that only commodity  $C$  is consumed, if the rate of profit  $r$  and the growth rate  $g$  are such that  $0 \leq r \leq R_1$  and  $0 \leq g \leq g^*$ , then processes (1)-(7) can be operated with the following intensities:

$$x_1 = \frac{5 + 16g + 19g^2 + 10g^3 + 2g^4}{5(25 + 18g - 19g^2 - 29g^3 - 13g^4 - 2g^5)} C^*$$

$$x_2 = 0$$

$$x_3 = \frac{1 - 2g - 3g^2 - g^3}{5(25 + 18g - 19g^2 - 29g^3 - 13g^4 - 2g^5)} C^*$$

$$x_4 = \frac{2 + g}{5(25 + 18g - 19g^2 - 29g^3 - 13g^4 - 2g^5)} C^*$$

$$x_5 = \frac{(2 + g)(1 + 2g + g^2)}{5(25 + 18g - 19g^2 - 29g^3 - 13g^4 - 2g^5)} C^*$$

$$x_6 = \frac{1 + 5g + 4g^2 + g^3}{5(25 + 18g - 19g^2 - 29g^3 - 13g^4 - 2g^5)} C^*$$

$$x_7 = 0$$

where  $C^*$  is the amount of commodity  $C$  available for consumption. Moreover, processes (1) and (3)-(6) determine the following prices, which are such that processes (2) and (7) cannot be operated since at these prices their costs are larger than their proceeds:

$$w = \frac{5(25 + 18r - 19r^2 - 29r^3 - 13r^4 - 2r^5)}{11 + 25r + 24r^2 + 11r^3 + 2r^4}$$

$$p_C = 1$$

$$p_{nm} = \frac{30(6 + 9r + 5r^2 + r^3)}{11 + 25r + 24r^2 + 11r^3 + 2r^4}$$

$$p_{M1} = \frac{30(4 + 8r + 5r^2 + r^3)}{11 + 25r + 24r^2 + 11r^3 + 2r^4}$$

$$p_{M2} = \frac{30(2 + 5r + 4r^2 + r^3)}{11 + 25r + 24r^2 + 11r^3 + 2r^4}$$

$$p_{N1} = \frac{30(3 + 6r + 4r^2 + r^3)}{11 + 25r + 24r^2 + 11r^3 + 2r^4}$$

Similarly, if  $0 \leq r \leq G$  and  $g^* \leq g \leq G$ , then processes (1)-(7) can be operated with the following intensities:

$$x_1 = \frac{5 + 16g + 19g^2 + 10g^3 + 2g^4}{5(27 + 14g - 25g^2 - 31g^3 - 13g^4 - 2g^5)} C^*$$

$$x_2 = \frac{-1 + 2g + 3g^2 + g^3}{5(27 + 14g - 25g^2 - 31g^3 - 13g^4 - 2g^5)} C^*$$

$$x_3 = 0$$

$$x_4 = \frac{2 + g}{5(27 + 14g - 25g^2 - 31g^3 - 13g^4 - 2g^5)} C^*$$

$$x_5 = \frac{3 + 3g + g^2}{5(27 + 14g - 25g^2 - 31g^3 - 13g^4 - 2g^5)} C^*$$

$$x_6 = \frac{(1 + g)(2 + g)}{5(27 + 14g - 25g^2 - 31g^3 - 13g^4 - 2g^5)} C^*$$

$$x_7 = 0.$$

Moreover, processes (1)-(2) and (4)-(6) determine the following prices, which are such that processes (3) and (7) cannot be operated since at these prices their costs are larger than their proceeds:

$$w = \frac{5(27 + 14r - 25r^2 - 31r^3 - 13r^4 - 2r^5)}{11 + 25r + 24r^2 + 11r^3 + 2r^4}$$

$$p_C = 1$$

$$p_{nm} = \frac{10(19 + 25r + 12r^2 + 2r^3)}{11 + 25r + 24r^2 + 11r^3 + 2r^4}$$

$$p_{M1} = \frac{10(20 + 36r + 21r^2 + 4r^3)}{11 + 25r + 24r^2 + 11r^3 + 2r^4}$$

$$p_{M2} = \frac{10(10 + 23r + 17r^2 + 4r^3)}{11 + 25r + 24r^2 + 11r^3 + 2r^4}$$

$$p_{N1} = \frac{10(15 + 27r + 17r^2 + 4r^3)}{11 + 25r + 24r^2 + 11r^3 + 2r^4} .$$

Finally, if  $g = g^*$  and  $0 \leq r < R_1$ , only processes (1) and (4)-(6) are operated: prices and the wage rate are not determined, and they may vary in a range.

## References

Betancourt, R. R. and C.K. Clague, 1981. Capital Utilisation. A Theoretical and Empirical Analysis (Cambridge University Press, Cambridge).

- Dos Santos Ferreira, R., 1974. Valeur et agrégation dans la problématique de la production, Ph.D. thesis (Université Louis Pasteur, Strasbourg, France).
- Fisher, F. M., 1993. Aggregation. Aggregate production functions and related topics, edited by J. Monz (The MIT Press, Cambridge, MA).
- Foss, M. F., 1981. Long-run changes in the workweek of fixed capital. *American Economic Review, Papers and Proceedings*, 71, 58-63.
- Georgescu-Roegen, N., 1965. Process in farming versus process in manufacturing: a problem of balanced development. In: Papi, U., Nunn, Ch. (Eds), *Economic Problems of Agriculture in Industrial Societies*. Macmillan, London, pp. 497-528. Reprinted in Georgescu-Roegen, 1976, pp. 71-102.
- Georgescu-Roegen, G., 1966. *Analytical Economics. Issues and Problems* (Harvard University Press, Cambridge, MA).
- Georgescu-Roegen, N., 1970. The economics of production. The Richard T. Ely Lecture. *American Economic Review* 60, 1-9. Reprinted in Georgescu-Roegen, 1976, pp. 61-70.
- Georgescu-Roegen, G., 1971. *The Entropy Law and the Economic Process* (Harvard University Press, Cambridge, MA).
- Georgescu-Roegen, N., 1976. *Energy and Economic Myths* (Pergamon Press, New York).
- Georgescu-Roegen, N., 1992. Georgescu-Roegen about himself. In: Szenberg, M., (Ed.), *Eminent economists: their life philosophies*. Cambridge University Press, Cambridge, pp. 128-160.
- Kurz, H. D., 1990, *Capital, Distribution and Effective Demand* (Polity Press and Basil Blackwell, Cambridge).
- Kurz, H. D., 1998. Against the current: Sraffa's unpublished manuscripts and the history of economic thought. *European Journal of the History of Economic Thought*, 5, 437-451.
- Kurz, H. D., and N. Salvadori, 1993. Von Neumann's growth model and the "classical" tradition. *European Journal of the History of Economic Thought*, 1, 129-160.

- Kurz, H. D., and N. Salvadori, 1995, *Theory of Production. A Long-Period Analysis* (Cambridge University Press, Cambridge).
- Kurz, H. D., and N. Salvadori, 1998. Reverse capital deepening and the numeraire: a note. *Review of Political Economy*, 10, 415-426.
- Kurz, H. D., and N. Salvadori, 2000. Economic dynamics in a simple model with exhaustible resources and a given real wage rate. *Structural Change and Economic Dynamics*, 11, 167-179.
- Kurz, H. D., and N. Salvadori, 2001. Classical economics and the problem of exhaustible resources. *Metroeconomica*, 52, 282-296.
- Maneschi, A., and S. Zamagni, 1997. Nicholas Georgescu-Roegen, 1906-1994. *Economic Journal*, 107, 695-707.
- Mill, J. S., 1965. *Principles of Political Economy with Some of Their Applications to Social Philosophy* (1st edn 1848). Edited by J. M. Robson, University of Toronto Press, Toronto.
- Neumann, J. von, 1945. A model of general economic equilibrium. *Review of Economic Studies*, 13, 1-9. Originally published in German in 1937.
- Piacentini, P., 1995. A time-explicit theory of production: analytical and operational suggestions following a "fund-flow" approach. *Structural Change and Economic Dynamics*, 6, 461-483.
- Ricardo, D., 1951. *On the Principles of Political Economy and Taxation* (1st edn 1817, 3rd edn 1821). In: *The Works and Correspondence of David Ricardo*, edited by P. Sraffa with the collaboration of M. H. Dobb, Vol. I, Cambridge University Press, Cambridge.
- Salvadori, N., 1999. Transferable machines with uniform efficiency paths. In: G. Mongiovi and F. Petri, (Eds.), *Value, Distribution and Capital*. Routledge, London and New York, pp. 297-313.
- Samuelson, P. A., 1962. Parable and realism in capital theory: The surrogate production function. *Review of Economic Studies*, 29, 193-206.
- Sandelin, B., 1976. On the origin of the Cobb Douglas production function. *Economy and History*, 19, 117-123.

- Sraffa, P., 1960. *Production of Commodities by Means of Commodities* (Cambridge University Press, Cambridge).
- Wicksell, K., 1893. *Über Wert, Kapital und Rente nach den neueren nationalökonomischen Theorien* (Gustav Fischer, Jena).
- Wicksteed, P. H., 1992. *The Co-ordination of the Laws of Distribution*, first published in 1894. Revised edition with an introduction by Ian Steedman (Aldershot: Edward Elgar).
- Winston, G. C., 1982. *The Timing of Economic Activities* (Cambridge University Press, Cambridge).