Production, Prices and Time: A Comparison of Some Alternative Concepts

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ABSTRACT: The paper compares alternative concepts of production and prices with special emphasis on time. First it is demonstrated that the point input-point output representation of processes used in von Neumann-Sraffa models is not restrictive and can be derived from general flow input-flow output processes. Second, the concept of long-period positions, which can be traced back to the work of the classical authors, is discussed. Third, the von Neumann-Sraffa approach is compared to the neo-Austrian model and to the flow-fund model developed by Hicks and Georgescu-Roegen respectively. It turns out that these latter two models are, at best, a special case of the former. Finally, some problems and intricacies concerning observable input-output coefficients are discussed and, as an alternative, a possibly applicable method to determine coefficients for general flow input-flow output processes is presented.

KEYWORDS: Classical economic theory; Sraffa; von Neumann; (neo)Austrian production theory; Georgescu-Roegen´s flow-fund approach; input-output coefficients

1. Introduction

This paper aims at a comparison of some concepts in the theory of production, accumulation and distribution which can be found in the writings of John Hicks, Piero Sraffa, John von Neumann, Wassily Leontief and Nicholas Georgescu-Roegen. This is not an easy task because these authors were developing their concepts and tools with different backgrounds and with different intentions.

Sraffa (1960) was primarily concerned with a revival of the concepts of the old classical economists; his investigation is concerned exclusively with economic systems which are in a selfreplacing state.

Seeking for an equilibrium solution of an economic system expanding with a uniform rate, von Neumann (1937, 1945) had to define all scarce resources away and postulate constant

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returns. In order to ensure non negative prices he assumed the rule of free goods. Neither the former nor the latter assumption can be found in Sraffa (1960). Scrutinising the work of both authors one will, however, find some unifying concepts and methods which are (i) the concept of a 'long run position', (ii) the view that production is a circular process where commodities are produced by means of commodities, and (iii) a totally vertically disintegrated representation of production by defining point input - point output processes with uniform duration.

Hicks (1970, 1973) proposed the other extreme: a totally vertically integrated representation of production. His approach, labelled 'neo-Austrian', is based on the 'Austrian theory of capital' founded by Böhm-Bawerk (1889) and elaborated by Wicksell (1934). The 'Austrians' were primarily interested in the time structure of production and conceived of a process as a series of dated original factor services which, finally, produce a quantity of a consumer good. Hence, production is viewed as a one way avenue leading from primary inputs to consumption goods.

With regard to the extremes of total vertical integration or disintegration, Georgescu-Roegen takes an intermediate position. He defines elementary processes by flows of inputs and by flows of outputs. Factors which are used up during the process are considered as 'flow-elements'. Other factors, such as land, labour, buildings or machines, are utilized but are not used up. These factors are labelled 'fund-elements'. Scrutinizing the flow-fund approach proposed by Georgescu-Roegen (1971, 1976) it is hard to find any restrictive assumptions, except with regard to durable capital and joint production. It is, however, also hard to find any operational conclusions.

In contrast to the scholars mentioned above, Leontief was primarily concerned with the empirical implementation and the application of multisectoral models of production. Therefore his models, as well as input-output models in general, are based on restrictive assumptions concerning joint production, natural resources, the treatment of durable capital, and, in particular, the time profile of inputs and outputs.

In Section 2 some basic concepts of production theory are expounded and it is demonstrated that, once some general propositions are accepted, the representation of technological possibilities by matrices for inputs and outputs is not restrictive at all if joint production is permitted. Section 3 starts with a general discussion of prices and related notions such as discount factors and commodity rates and proceeds with the definition of equilibrium, and, in particular, with the concept of long-run positions. It is demonstrated that in equilibrium the dates of payments are irrelevant. Furthermore, the choice of the numeraire has no influence on the rate of profit if, and only if, we can assume that (undiscounted) prices are constant. Section 4 provides a short account of methods, concepts and some conclusions which can be found in the writings of modern classical economists such as von Neumann or Sraffa. Section 5 is concerned with the Austrian view of production and, in particular, with the neo-Austrian concept proposed by Hicks. These concepts appear as special cases of a von Neumann - Sraffa representation of production. Furthermore, if there is more than one Austrian process, and if these processes are not based on a von Neumann - Sraffa technique, then results obtained are incompatible with the notion of equilibrium. In section 6 Georgescu-Roegen's flow-fund approach is discussed. The emphasis is on the concept of fixed capital and on the treatment of waste. It turns out that Georgescu-Roegen's proposal involves assumptions and propositions which are unnecessarily restrictive. Finally, in section 7, the concept of flow-flow coefficients and stock-flow ratios are studied. It is demonstrated that these coefficients, which provide the basis for input-output models, depend, in general, on the intensity and on the dynamics of production, and can, therefore, not be taken as 'technical' coefficients.

The present paper benefits from previous efforts which have been made to compare different concepts to the theory of production and prices. For a comprehensive discussion of similarities and differences between von Neumann and Srafa see Kurz and Salvadori (1995, pp. 403-426). For a comparison of the Sraffa - von Neumann approach with the 'neo-Austrian' model proposed by Hicks see Burmeister (1974) and Hagemann and Kurz (1976). A reformulation of Georgescu-Roegen´s flow-fund model within a multisectoral framework has been provided by Tani (1988). A critical discussion of the flow-fund approach and a comparison with the approach of the classical economists can be found in Kurz and Salvadori (1999).

2. Production and time: some basic concepts

Explaining the activity analysis model adopted by Sraffa and by von Neumann one usually starts as follows: There are m processes which use produced means of production and homogenous labour to produce n commodities. It is assumed that all processes are of unit time duration. A process k can be described as

$$(1)\qquad \left\{ \begin{pmatrix} \mathbf{a}^k \\ l^k \end{pmatrix} \to \mathbf{b}^k \right\},$$

where row vectors $\mathbf{a}^k, \mathbf{b}^k \in R^n$ refer to inputs and outputs of n products and l^k is a scalar of quantities of homogenous labour.

With the exception of the 'assumption' that there is a uniform period of production, there is no reference to time. In particular, the time profile of production, i.e. the dates when inputs are used and outputs are produced, seems to be overly simplified. In order to demonstrate that the Sraffa - von Neumann approach is much more general than it seems at a first glance we will start with a more general description of a process and look how that fits into the Sraffa - von Neumann approach. We may start from the following propositions:

(P.1) Production requires time.

(P.2) Production requires inputs, i.e. some producible and/or some non producible factors (*no land of cockaigne*).

(P.3) It is not possible to transform inputs into nothing (*no free disposal*).

(P.4) All firms have access to all known methods of production, and, in particular, the set of methods of production available to each single firm is independent of the size of the firm itself (*constant returns within the single firm*).

(P.5) Two equal quantities of two products or factors are considered as being equal if they are perfect substitutes for all producers and all consumers; otherwise they are different.

(P.6) There is a *finite* number of different products and a *finite* number of different factors differentiated with respect to quality, location and time.

(P.7) There is a finite number of production processes.

Propositions P.1, P.2 and P.3 are obvious consequences of fundamental physical principles. Instantaneous production as well as production without any inputs is nonsense and is therefore ruled out by P.1 and P.2 respectively. P.3 rules out pure abatement processes which use some inputs and absorb some unwanted products but produce nothing. At the very end of any process there is some product. Its value may be non positive (zero or negative). Hence this proposition does neither exclude the possible production of 'bads' nor the existence of produced goods which are free.

Proposition P.4 is crucial for free competition and is, therefore, a necessary precondition for the existence of a competitive equilibrium (cf. Kurz and Salvadori, 1995, p. 17). Note that this proposition does neither postulate that all processes are perfectly divisible nor suppose that all processes are additive. Constant returns to scale within each single firm does not rule out diminishing or increasing returns external to firms, that is the case in which the size of one or several industrial sectors determine the set of available processes. It is, for instance, not the single farming process but it is the agricultural sector as a whole which is constrained by the scarcity of land, and it is not a particular firm but the whole manufacturing sector which benefits from the increasing division of labour induced by growing markets.

Propositions P.5 and P.6 refer to factors and products. We may distinguish non producible (natural) factors and produced means of production (capital goods). Natural factors may be further subdivided into the broader categories 'human labour', 'non exhaustible resources', such as Ricardian land or the energy delivered by the sun, 'depletable resources', such as crude oil or coal in the ground, and 'renewable natural resources', such as virgin forests or natural fish populations. Following an old tradition we may classify produced means of production according to their durability under the heads of circulating and fixed capital. The former refer to inputs, such as raw materials or energy carriers, which are used up. Fixed capital items, such as machines or buildings, are durable and contribute to the production of a flow of outputs. Therefore, the value of fixed capital is, in contrast to circulating capital, never fully recovered in one unit of the product.

The question 'What is a product?' cannot be given independently of the scope of the analysis. Engineers interested in a blast furnace process will consider the amount of liquid steel produced as a considerable category. Economists are interested first and foremost in *exchanged* products. Hence we might consider only those outcomes of a process as products proper which are utilized in other processes or are used as consumer goods. But output of production is not always useful. Some products may impede other producers or create harm to households and must be taken into account. Hence we must consider all those outcomes of a process as products which leave the process and enter its environment. Consequently the definition of a product depends crucially on the definition what is a process.

Proposition (P.5) is the usual condition for distinction of products and factors. There are three characteristics of differentiation. (i) quality: the physical nature and attributes, which determine the way in which products and factors meet the needs of the users; (ii) location: two equal quantities of the same product or factor are not perfect substitutes if they are not available in the same place. Hence we consider two goods available in different locations as two distinct goods and the transport from the first place to the second as a process of production with the first good as input and the second good as output; (iii) time: two equal quantities of the same product or the same factor being available at different dates are not equivalent but are considered as being different. Note that by defining a process as an activity which transforms factors into products and given these characteristics of differentiation we have included in our definition of a process not only processes of production but also separable transport and storage activities.

Proposition P.6 is crucial as it implies that quality, location and, in particular, time are *discretely* measurable variables. If the range of qualities of some products appears perfectly continuous we would have an infinite number of goods. But in actual fact, commodities which show a wide range of qualities are classified into a few different groups. The same argument holds, in principle, also for location and time.

Taking account of the fact that the capacities of fixed capital in place could be utilized in many, perhaps in infinite many ways, the possibility of an infinite number of processes can not be neglected. Therefore, (P.7) is restrictive, but, however, not more restrictive than to postulate an infinite number of production methods.

Given these propositions and assuming that there are n products and u original factors differentiated with respect to quality and location we may describe a process k by

$$\text{(2)} \qquad \left(\begin{pmatrix}\mathbf{a}_0^k\\ \mathbf{c}_0^k\end{pmatrix},\begin{pmatrix}\mathbf{a}_1^k\\ \mathbf{c}_1^k\end{pmatrix},\cdots\begin{pmatrix}\mathbf{a}_{T_k}^k\\ \mathbf{c}_{T_k}^k\end{pmatrix}\right)\rightarrow\left(\mathbf{b}_0^k,\mathbf{b}_1^k,\cdots\mathbf{b}_{T_k}^k\right),$$

i.e. a flow of inputs and a flow of outputs of finite duration, where $\mathbf{a}_t^k,\mathbf{b}_t^k \in R^n$ are vectors of inputs and outputs of finished products at time t, $\mathbf{c}_t^k \in R^u$ are vectors of inputs of original factors and T_k is the duration of process k.

From proposition (P.1) to (P.3) it follows that

(i) $\quad T_k > 0$;

(ii) $\quad \begin{pmatrix}\mathbf{a}_0^k\\ \mathbf{c}_0^k\end{pmatrix} \geq \mathbf{0}; \qquad \mathbf{b}_t^k = \mathbf{0}$ for $\forall t:\ 0 \leq t \leq t_k$, where $t_k:\ 0 \leq t_k < T_k$

(iii) $\quad \begin{pmatrix}\mathbf{a}_{T_k}^k\\ \mathbf{c}_{T_k}^k\end{pmatrix} = \mathbf{0}; \qquad \mathbf{b}_{T_k}^k \geq \mathbf{0}$;

(i) and (ii) are immediate consequences of (P.1) and (P.2): each process is initiated by the use of some inputs. Hence, at least one element of vectors $\mathbf{a}_0^k$ or $\mathbf{c}_0^k$, is positive. Since any transformation of inputs into outputs requires time the first quantities of outputs can emerge only after a certain stretch of time has elapsed.

Is it possible that at the end of production time some inputs are used and no output appears? Because it is not possible to transform inputs into nothing (P.3), the answer is: no. This does not rule out the possibility of 'shut down' or 'clean up' activities which may be required at the end of some productive process. If, for example, some shut-down activities are necessary to make a nuclear power plant more or less harmless, then the output of these activities is a closed and more or less harmless nuclear power plant. Its price may be zero or negative but there is, however, an existing output. Hence $\mathbf{b}_{T_k}^k \geq \mathbf{0}$. Furthermore, from the fact that the transformation of inputs into outputs requires time it follows that $\mathbf{a}_{T_k}^k = \mathbf{0}$ and $\mathbf{c}_{T_k}^k = \mathbf{0}$.

Note that the definition of a process involves its duration. Hence, a feasible *truncation* of process k, i.e. a process u where $T_u < T_k$, $\mathbf{a}_t^u = \mathbf{a}_t^k$, $\mathbf{c}_t^u = \mathbf{c}_t^k$, $\mathbf{b}_t^u = \mathbf{b}_t^k$, $\forall t: \ t \leq T_u$ and $\mathbf{b}_{T_u}^u \geqq \mathbf{b}_{T_u}^k \geq \mathbf{0}$, is considered as a different process. Note that permitting all truncations, even those where $\mathbf{b}_{T_u}^u = \mathbf{b}_{T_u}^k = \mathbf{0}$, would violate proposition (P.3).

Without loss of generality, we may neglect the vectors $\mathbf{a}_{T_k}^k$, $\mathbf{c}_{T_k}^k$ and $\mathbf{b}_0^k$, which contain zero elements only, and represent a general flow input-flow output process by

(3) $$\left(\begin{pmatrix} \mathbf{a}_0^k \\ \mathbf{c}_0^k \end{pmatrix}, \begin{pmatrix} \mathbf{a}_1^k \\ \mathbf{c}_1^k \end{pmatrix}, \cdots \begin{pmatrix} \mathbf{a}_{T_k-1}^k \\ \mathbf{c}_{T_k-1}^k \end{pmatrix} \right) \rightarrow \left(\mathbf{b}_1^k, \mathbf{b}_2^k, \cdots \mathbf{b}_{T_k}^k \right).$$

The following special cases of general flow input-flow output processes may be considered:

- flow input-point output processes, where outputs appear exclusively at the end of production time, i.e. $\mathbf{b}_t^k = \mathbf{0}$ for $t < T_k$,

- point input-point output processes, where inputs and outputs appear exclusively at the beginning and at the end of the process, i.e. $\begin{pmatrix} \mathbf{a}_0^k \\ \mathbf{c}_0^k \end{pmatrix} \rightarrow \mathbf{b}_{T_k}^k$, or

- single production processes producing products which are homogenous with respect to quality (and location). Outputs of single production processes which produce amounts of commodity *j* are characterized by $\mathbf{b}_t^k = \mathbf{e}_j b_t^k$, $\forall t$, where $\mathbf{e}_j$ is a vector whose *j*-th element is equal to one and all other elements are equal to zero and $b_t^k \geq 0$ are scalars which represent the quantities of the output which is homogenous, except with respect to time. Hence, the only element of joint production involved is intertemporal joint production.

The definition of a flow input-flow output process involves some arbitrariness regarding the degree of vertical integration. The production of bread is usually seen as a bakery process where, among other inputs, flour is used, which is the product of a grinding process that uses corn. We may, however, take another view and merge both processes such that bread is produced by a vertically integrated grinding-bakery process which uses corn and produces bread. Inputs and outputs of flour have disappeared from the scene, and therefore, we can drop flour from our list of products. We might continue merging processes and thereby proceed in excluding some other commodities which appear as inputs and outputs from our list of products. In the limit of that procedure we define away all capital goods and obtain totally vertically integrated processes which use a flow of primary inputs to produce a flow of consumer goods. This approach will be discussed in section 5.

The other extreme consists of a totally vertically disintegrated representation of production. Any flow input-flow output process having a duration which exceeds the unit of time can be broken down into a finite number of point input - point output processes[1] of unit duration by introducing additional 'intermediate' goods which connect the time series of subsequent processes. Hence,

$$(4)\qquad \overbrace{\begin{pmatrix}\mathbf{a}_0^k\\ \mathbf{0}\\ \mathbf{c}_0^k\end{pmatrix}\rightarrow\begin{pmatrix}\mathbf{b}_1^k\\ \mathbf{e}_1^k\, l_1^k\end{pmatrix}}^{k_1};\ \overbrace{\begin{pmatrix}\mathbf{a}_1^k\\ \mathbf{e}_1^k\, l_1^k\\ \mathbf{c}_0^k\end{pmatrix}\rightarrow\begin{pmatrix}\mathbf{b}_2^k\\ \mathbf{e}_2^k\, l_2^k\end{pmatrix}}^{k_2};\cdots\ \overbrace{\begin{pmatrix}\mathbf{a}_{t-1}^k\\ \mathbf{e}_{t-1}^k\, l_{t-1}^k\\ \mathbf{c}_0^k\end{pmatrix}\rightarrow\begin{pmatrix}\mathbf{b}_t^k\\ \mathbf{e}_t^k\, l_t^k\end{pmatrix}}^{k_t};\cdots\ \overbrace{\begin{pmatrix}\mathbf{a}_{T_k-1}^k\\ \mathbf{e}_{T_k-1}^k\, l_{T_k-1}^k\\ \mathbf{c}_{T_k-1}^k\end{pmatrix}\rightarrow\begin{pmatrix}\mathbf{b}_{T_k}^k\\ \mathbf{0}\end{pmatrix}}^{k_{T_k}}$$

is an equivalent *vertically disintegrated* point input - point output representation of the general flow input - flow output process (3) which, on the other hand, can be conceived of as a *vertically integrated* representation of the set of point input - point output processes defined by (4). $\mathbf{e}_t^k$ are vectors of dimension $T_k - 1$ where the *t*-th elements are equal to one and all other elements are equal to zero. The scalar l_t^k represents the quantity of an intermediate good produced in process k_t and used up in process k_{t+1}. These intermediate goods can be conceived of as composite commodities representing all semifinished goods, all capital items under construction and all used fixed capital goods which are produced in process k_t and which are transferred to be used in the subsequent process k_{t+1}. Note that old machines which are used in process *k* and which are transferred to be used in another process are *exchanged*

[1] The discrete time representation is a necessary condition to avoid dealing with an infinite number of point input-point output processes.

products and are taken into account in vectors $\mathbf{b}_t^k$. Hence, if durable capital items used by process k are transferable, then outputs of that process are not homogenous with respect to quality, and, therefore, we cannot avoid to deal with general joint production.

Without loss of information, we may set all $l_t^k = 1$ and represent the T_k processes in compact matrix format:

$$(5) \quad \left(\begin{pmatrix} \mathbf{A}^{(k)} \\ \mathbf{M}^{(k)} \end{pmatrix}, \mathbf{C}^{(k)} \right) \rightarrow \begin{pmatrix} \mathbf{B}^{(k)} \\ \mathbf{N}^{(k)} \end{pmatrix},$$

where $\mathbf{A}^{(k)} = \begin{pmatrix} \mathbf{a}_0^k & \mathbf{a}_1^k & \cdots & \mathbf{a}_{T_k-1}^k \end{pmatrix} \in \mathbb{R}_+^{n \times T_k}$, $\mathbf{B}^{(k)} = \begin{pmatrix} \mathbf{b}_1^k & \mathbf{b}_2^k & \cdots & \mathbf{b}_{T_k}^k \end{pmatrix} \in \mathbb{R}_+^{n \times T_k}$, refer to inputs and outputs of finished products, $\mathbf{M}^{(k)} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \end{pmatrix} \in \mathbb{R}_+^{(T_k-1) \times T_k}$, $\mathbf{N}^{(k)} = \begin{pmatrix} \mathbf{I} & \mathbf{0} \end{pmatrix} \in \mathbb{R}_+^{(T_k-1) \times T_k}$, are matrices for inputs and outputs of intermediate products, where the first (last) column is nought and **I** is a unit matrix of dimension $T_k - 1$. $\mathbf{C}^{(k)} = \begin{pmatrix} \mathbf{c}_0^k & \mathbf{c}_1^k & \cdots & \mathbf{c}_{T_k-1}^k \end{pmatrix} \in \mathbb{R}_+^{u \times T_k}$ refers to inputs of primary factors.

Assume that there are m flow input flow - output processes of duration T_k, $k = 1, 2, \cdots m$, by which n finished products are producible. Breaking down each process into processes of unit duration and introducing an intermediate product for each additional process, we end up with $\tilde{m} = \sum_{k=1}^{m} k \times T_k$ point input - point output processes which produce $\tilde{n} = n + \sum_{k=1}^{m} k \times (T_k - 1)$ products. Note that $(n = m \Rightarrow \tilde{n} = \tilde{m})$, i.e. if the number of flow input - flow output processes is equal to the number of finished products, then the number of the corresponding point input - point output processes equals the number of finished and intermediate products.

Defining matrices

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}^{(1)} & \mathbf{A}^{(2)} & \cdots & \mathbf{A}^{(m)} \\ \mathbf{M}^{(1)} & 0 & \cdots & 0 \\ 0 & \mathbf{M}^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{M}^{(m)} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \mathbf{B}^{(1)} & \mathbf{B}^{(2)} & \cdots & \mathbf{B}^{(m)} \\ \mathbf{N}^{(1)} & 0 & \cdots & 0 \\ 0 & \mathbf{N}^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{N}^{(m)} \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} \mathbf{C}^{(1)} & \mathbf{C}^{(2)} & \cdots & \mathbf{C}^{(m)} \end{pmatrix},$$

it is straightforward to represent any finite set of m flow input - flow output processes which produce n finished products by means of n finished products and u non-producible resources by the triplet $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$, where $\mathbf{A}, \mathbf{B} \in R_+^{\tilde{n} \times \tilde{m}}$ refer to matrices of inputs and outputs of finished and intermediate products and $\mathbf{C} \in R_{+}^{u \times \tilde{m}}$ is a matrix of non-producible factors. Provided one accepts the propositions and assumptions given above, the representation of technology by input and output matrices is not restrictive at all. For certain purposes one may, however, add some additional assumptions. Assuming, for instance, that all flow input - flow output processes are single production processes, we obtain a 'pure fixed capital system' which is a particular joint production system which retains all the nice properties of systems without any joint production. For a comprehensive discussion of such 'pure fixed capital systems', see Schefold (1989, pp. 145-197) or Kurz and Salvadori (1995, Chapter 7).

3. Prices and time: some basic concepts

A price of a product (factor) is, in general, the exchange value of that product (factor) in terms of a numeraire. Usually one particular product, a constant bundle of products or the unit of labour serve as numeraire, which might be called 'money'. Since two quantities are involved, i.e. the quantity of the product (factor) and the quantity of 'money' (denoted by q_t^i and q_T^m), which may both differ in quality (and location) and time of availability we may define the price of commodity *i* delivered at date *t* and paid at date *T* as

(6) $$p_{t,T}^{i,m} = \frac{q_T^m}{q_t^i},$$

which is equal to the amount of 'money' given at date *T* in exchange for one unit of good *i* delivered at date *t*.

In order to avoid confusion created by excessive notation, prices are usually defined either as

(7) $$\tilde{p}_t^i \equiv p_{t,0}^{i,m} = \frac{q_0^m}{q_t^i},$$

i.e. *discounted prices* of good *i* delivered at date *t* and paid at date 0 or as

(8) $$p_t^i \equiv p_{t,t}^{i,m} = \frac{q_t^m}{q_t^i},$$

i.e. *undiscounted prices* of good *i* delivered and paid at the same date *t*.

By defining the own discount factor of a good *i*, $p_{t,0}^{i,i} \equiv b_t^i = \frac{q_0^i}{q_t^i}$, and, in particular, the own discount factor of 'money', or, for short, the discount factor,

(9) $p_{t,0}^{m,m} = \tilde{p}_t^m = b_t = \dfrac{q_0^m}{q_t^m}$,

which is the amount of 'money' given at date 0 per unit of 'money' given at date t, discounted and undiscounted prices are related such that

(10) $\tilde{p}_t^i = b_T p_{t,T}^{i,m}$ and, in particular, $\tilde{p}_t^i = b_t p_t^i$.

We may define, furthermore, the own rate of interest of good i and, in particular, the (own) rate of interest (of 'money') for period $[t, t+1]$ which are the numbers r_t^i and r_t, respectively, such that

(11) $1 + r_t^i = \dfrac{b_t^i}{b_{t+1}^i} = \dfrac{q_{t+1}^i}{q_t^i}$,

(12) $1 + r_t = \dfrac{b_t}{b_{t+1}} = \dfrac{q_{t+1}^m}{q_t^m}$.

Note that the own discount factor of a good i (and its own rate of interest) and the discount factor of 'money' (and the 'money' rate of interest) are related such that

(13) $b_t^i = b_t \dfrac{p_t^i}{p_0^i}$,

(14) $1 + r_t^i = (1 + r_t) \dfrac{p_t^i}{p_{t+1}^i}$.

Assume, for simplicity, that commodities are produced by means of commodities and by homogenous labour. A flow input-flow output process k, operated at intensity x^k, is determined by $\left(\begin{pmatrix} \mathbf{a}_0^k \\ l_0^k \end{pmatrix}, \begin{pmatrix} \mathbf{a}_1^k \\ l_1^k \end{pmatrix}, \cdots \begin{pmatrix} \mathbf{a}_{T_k-1}^k \\ l_{T_k-1}^k \end{pmatrix} \right) x^k \rightarrow \left(\mathbf{b}_1^k, \mathbf{b}_2^k, \cdots \mathbf{b}_{T_k}^k \right) x^k$, where vectors $\mathbf{a}_t^k, \mathbf{b}_t^k \in \mathbb{R}_+^n$ with elements a_t^{ik} and b_t^{jk} represent flows of inputs and outputs of products, and scalars l_t^k are inputs of homogenous labour.

If all operated (i.e. cost-minimizing) processes exhibit neither extra profits nor extra costs then the economy is said to be in equilibrium. Hence, for all operated processes the present value of costs equals the present value of revenues, i.e.

(15) $\forall k : x^k > 0 \quad \Rightarrow \quad \sum_{i=1}^{n} \sum_{t=0}^{T_k -1} a_t^{ik} \, p_{t,t_i}^{i} \, b_{t_i} + w_{t,t_l} \, b_{t_l} \, l_t^{ik} = \sum_{j=1}^{n} \sum_{t=1}^{T_k} b_t^{jk} \, p_{t,t_j}^{j} \, b_{t_j}$,

where p_{t,t_j}^{j} is the price of good *j* delivered at time *t* and paid at time t_j, and w_{t,t_l} is the wage rate for labour used during the interval [*t*, *t*+1] and paid at time t_l.

Recalling the definitions for discounted and undiscounted prices given above, we obtain immediately that $p_{t,t_i}^{i} = \tilde{p}_t^{i} \dfrac{1}{b_{t_i}} = p_t^{i} \dfrac{b_t}{b_{t_i}}$ and $w_{t,t_l} = \tilde{w}_t \dfrac{1}{b_{t_l}} = w_t \dfrac{b_t}{b_{t_l}}$.

Hence we may conveniently specify condition (15) either in terms of discounted prices, i.e.

(16) $\forall k : x^k > 0 \quad \Rightarrow \quad \sum_{t=0}^{T_k -1} \tilde{\mathbf{p}}_t \mathbf{a}_t^k + \tilde{w}_t l_t^k = \sum_{t=1}^{T_k} \tilde{\mathbf{p}}_t \mathbf{b}_t^k$,

or in terms of undiscounted prices, i.e.

(17) $\forall k : x^k > 0 \quad \Rightarrow \quad \sum_{t=0}^{T_k -1} b_t \left(\mathbf{p}_t \mathbf{a}_t^k + w_t l_t^k \right) = \sum_{t=1}^{T_k} b_t \mathbf{p}_t \mathbf{b}_t^k$,

Note that (16) as well as (17) are independent of the dates of payment.

4. The Classical long-period approach

Scholars working in the tradition of the classical economists use the method of long-period positions. This method abstracts from any influence which is of a temporary or accidental nature and concentrates on the persistent forces governing a competitive market economy. As a consequence of the long-period perspective, the prices of produced commodities just cover the cost of production and, in order to guarantee reproduction, allow for profits at a 'normal' rate of return on the value of capital advanced at the beginning of the period of production.

The various classical contributions to the theory of prices, production and distribution bear some common features (cf. Kurz and Salvadori, 1995, p. 14). Given '*the set of data*' i.e.

(i) the technical conditions of production,
(ii) the volume and the composition of the social product,
(iii) the wage rate (or the rate of profits) and
(iv) the quantities of different qualities of land

and assuming that each firm has access to each method of production (cf. proposition P.4), then the following unknowns are determined:

(1) the cost-minimizing methods of production including the activity-levels of the various processes in use and the gross output of commodities produced,
(2) the prices of production,
(3) the rate of profit (or the wage rate) and
(4) the rate(s) of rent on different qualities of land.

The classical approach to economic change proceeds in terms of comparison of long-period positions of the economy. A change in the set of data (i)-(iv) will generate new results (1)-(4) i.e. the new normal position will emerge as a result of the new static conditions. If, for instance, technical progress makes new methods of production available, or if there is a change in the distributive variables, then excess profits in some sectors and subnormal profits in other sectors will generate incentives for capitalists to increase capacities in the first group of sectors and reduce capacities in other sectors. In the long run the quantity and the structure of capital goods will adjust to the new situation and thus a new long-run position will result. Once the new position is established, there exist no endogenous forces to change the prices of production. Hence, long-period positions are characterized by stationary prices. In order to exhibit this property, a classical model requires the following additional assumptions:

(C.1) A long run position is characterised by a given and constant set of production methods.

(C.2) A long run position is characterised by a given and constant distribution of income.

(C.3) If there is general joint production, then the economy is, in general, taken to be stationary or to grow in constant proportions.

(C.4) If there are decreasing or increasing returns to scale (which are external to the firm), then the economy is taken to be stationary.

(C.5) Exhaustible natural factors are set aside.

Given these assumptions which assure that stationary prices prevail, condition (15) can be written as

(18) $\forall k : x^k > 0 :$ $$\mathbf{p}\sum_{t=1}^{T_k} \mathbf{a}_t^k \,(1+r)^{1-t} + w\sum_{t=1}^{T_k} l_t^k\,(1+r)^{1-t} = \mathbf{p}\sum_{t=1}^{T_k} \mathbf{b}_t^k\,(1+r)^{1-t} \,.$$

It is easily checked that, by introducing new variables $v_t^k, \quad t = 1,2,\cdots,T_k - 1$, representing values of intermediate products, the equation in condition (18) can equivalently be represented by the vertically disintegrated price system of T_k point input-point output processes

(19.1) $$\left(\mathbf{p}\,\mathbf{a}_0^k + w\,l_0^k\right)(1+r) = \begin{pmatrix} \mathbf{p} \\ v_1^k \end{pmatrix}\begin{pmatrix} \mathbf{b}_1^k \\ 1 \end{pmatrix}$$

(19.2) $$\left(\begin{pmatrix} \mathbf{p} \\ v_1^k \end{pmatrix}\begin{pmatrix} \mathbf{a}_1^k \\ 1 \end{pmatrix} + wl_1^k\right)(1+r) = \begin{pmatrix} \mathbf{p} \\ v_2^k \end{pmatrix}\begin{pmatrix} \mathbf{b}_2^k \\ 1 \end{pmatrix}$$

$$\vdots$$

($19.T_k$) $$\left(\begin{pmatrix} \mathbf{p} \\ v_{T_k-1}^k \end{pmatrix}\begin{pmatrix} \mathbf{a}_{T_k-1}^k \\ 1 \end{pmatrix} + wl_{T_k-1}^k\right)(1+r) = \mathbf{p}\mathbf{b}_{T_k}^k$$

Obviously, the value of an intermediate product produced by process k_t and used by process k_{t+1}, $t = 1, 2, \cdots T-1$, is

(20) $$v_t^k = \frac{y_{t+1}^k + v_{t+1}^k}{1+r},$$

where

(21) $$y_{t+1}^k = \mathbf{p}\mathbf{b}_{t+1}^k - (1+r)\left(\mathbf{p}\mathbf{a}_t^k + wl_t^k\right)$$

is the net receipt that can be obtained by activating process k_{t+1}. It follows that the value of an intermediate product produced by process k_t and used by process k_{t+1} is equal to the capital value one gets by discounting all future net receipts that may be obtained by activating processes $k_{t+1}, k_{t+2}, \cdots k_T$, i.e.

(22) $$v_t^k = \sum_{t=1}^{T_k - t} y_{t+t}^k \,(1+r)^{-t}, \quad t = 1, 2, \cdots, T_k - 1,$$

Assume, for simplicity, that process k uses only one type of fixed capital good, say a machine. Then v_t^k represents the value of a t-year old machine. Hence, $d_t^k = v_t^k - v_{t+1}^k$ is the depreciation of fixed capital during period [t, t+1]. This demonstrates that the pattern of depreciation cannot be assumed by the analyst, but is endogenously determined and depend on technology and the distribution of income. For a comprehensive discussion of fixed capital models which take account of joint utilization of different types of fixed capital inputs, see Kurz and Salvadori (1995, chapters 7 and 9).

Assume that all flow input - flow output processes are single production processes, each of them producing a flow of products being homogenous with respect to quality and location. Thus, allowing for intertemporal joint production, and therefore permitting production with durable capital, but avoiding general joint production, we may define equation (18) in terms of discounted inputs per unit of discounted outputs and obtain

$$(23) \qquad \left(\mathbf{p}\,\tilde{\mathbf{a}}^k(r) + w\tilde{l}^k(r)\right)(1+r) = p_k ,$$

where the 'centre coefficients'

$$(24) \qquad \tilde{\mathbf{a}}^k(r) = \left(\sum_{t=1}^{T_k} b_t^k (1+r)^{1-t}\right)^{-1} \sum_{t=0}^{T_k-1} \mathbf{a}_t^k (1+r)^{-t}$$

$$(25) \qquad \tilde{l}^k(r) = \left(\sum_{t=1}^{T_k} b_t^k (1+r)^{1-t}\right)^{-1} \sum_{t=0}^{T_k-1} l_t^k (1+r)^{-t} ,$$

for capital and labour inputs are functions of the rate of profit.[2]

Given n single production flow input - flow output processes producing n finished products we obtain a 'pure fixed capital system' defined by $\left\{\tilde{\mathbf{A}}(r), \tilde{\mathbf{l}}(r)\right\}$, i.e. a square matrix of centre coefficients for capital inputs and a vector of centre coefficients for labour. Given the rate of profit and specifing a numeraire which may be either labour $(w = 1)$, or a bundle of products $(\mathbf{pd} = 1)$, the prices of finished goods are determined by

$$(26) \qquad \left(\mathbf{p}\,\tilde{\mathbf{A}}(r) + w\tilde{\mathbf{l}}(r)\right)(1+r) = \mathbf{p} .$$

It is easy to demonstrate[3], that if the system of production is viable (that is the system is capable to produce a surplus), i.e. if there exists a vector of intensities $\mathbf{x} \geq \mathbf{0}$, such that

$$(27) \qquad \left(\mathbf{I} - \tilde{\mathbf{A}}(0)\right)\mathbf{x} = \mathbf{f} \geq \mathbf{0} ,$$

then there exists a positive maximal rate of profit, R, and a non-negative vector $\mathbf{p}^*$ such that

$$(28) \qquad \mathbf{p}^*\left(\mathbf{I} - \tilde{\mathbf{A}}(R)(1+R)\right) = \mathbf{0} ,$$

and for all rates of profit $r:\ 0 \leq r < R$

$$(29) \qquad \left(\mathbf{I} - \tilde{\mathbf{A}}(r)(1+r)\right)^{-1} \geq \mathbf{0} .$$

Hence, prices for finished products in terms of labour commanded, i.e.

[2] The concept of 'centre coefficients' was introduced by Schefold (1971)
[3] See, for instance, Schefold (1989, pp. 155-157).

(30) $w = 1, \quad \mathbf{p} = \tilde{\mathbf{l}}(r)\left(\mathbf{I} - \tilde{\mathbf{A}}(r)(1+r)\right)^{-1} \geq \mathbf{0}$.

are positive for $r : 0 \leq r < R$, and are rising functions of the rate of profit, tending to infinity as the profit rate approaches *R*. Therefore the real wage rate, in terms of bundle **d**, is a decreasing function of the rate of profit such that

(31) $$w = \frac{1}{\tilde{\mathbf{l}}(r)\left(\mathbf{I} - \tilde{\mathbf{A}}(r)(1+r)\right)^{-1}\mathbf{d}} \geq \mathbf{0}, \quad \text{for } r: \quad 0 \leq r \leq R \ .$$

Note that in this case the *non-substitution theorem* holds, i.e. prices can be determined without reference to demand. This and other nice properties need not hold in the presence of joint production.

The validity of the non-substitution theorem does not mean that there is no substitution in production. In general, there will be more than one flow input-flow output process for each finished product. Hence we obtain more than one possible system of production. In the long run it is the *cost-minimizing system* which will prevail.

Deriving the cost-minimizing prices of production we may use either the direct or the indirect approach. The direct approach goes back to von Neumann (1937) and determines the cost-minimizing processes and the corresponding system of prices by solving a system of (in)equalities. The indirect approach, proposed by Sraffa (1960), is a stepwise procedure and consists in a comparison of square systems of production. A comprehensive discussion of the direct and the indirect approach can be found in Kurz and Salvadori (1995).

5. Subsectors, reduction to dated quantities of labour and the (neo-) Austrian concept of production

In the preceding sections it has been demonstrated, that any flow input-flow output process can be subdivided into several point input-point output processes. Setting aside primary inputs other than homogenous labour, a square technique can be characterized by a set of point input-point output processes represented by $\{\mathbf{A}, \mathbf{l}, \mathbf{B}\}$ where **A** and **B** are square matrices of inputs and outputs and **l** is a vector of labour inputs. Assume that the system can produce given net products and is cost-minimizing at a rate of profit *r* and at a wage rate *w* paid ante factum, then prices of production are determined by

(32) $\mathbf{p} = \mathbf{p}\mathbf{A}(1+r) + w\mathbf{l}$

There are some other ways to represent the technology (see Kurz and Salvadori, 1995, pp. 164-180). Using the concept of 'sub-systems', introduced by Sraffa (1960, Appendix A) and further developed by Pasinetti (1973), one may characterize a technique also by $\{\mathbf{A}^*, \mathbf{l}^*\}$, where $\mathbf{l}^* = \mathbf{l}(\mathbf{B} - \mathbf{A})^{-1}$ and $\mathbf{A}^* = \mathbf{A}(\mathbf{B} - \mathbf{A})^{-1}$ represent inputs of *vertically integrated* labour and vertically integrated capital goods. Whereas the elements of $\mathbf{A}$ and $\mathbf{l}$ refer to direct inputs, the elements of $\mathbf{A}^*$ and $\mathbf{l}^*$ represent total, i.e. direct and indirect inputs. The concept of vertical integration can be far traced back in the history of economic thought. It has been clearly spelled out by Adam Smith who put forward the idea that the value, or price, of any product can be expressed as a sum of three components: (i) wages, (ii) profits, and (iii) rents (here rents are set aside). Equation (32) may in fact be rewritten in terms of vertically integrated sectors, i.e.

(33) $\mathbf{p} = w\mathbf{l}^* + r\mathbf{p}\mathbf{A}^*$.

Setting aside joint production, i.e. assuming $\mathbf{B} = \mathbf{I}$, we may represent technology also by a series of *dated labour*, $\{\mathbf{l}_0, \mathbf{l}_1, \mathbf{l}_2, \cdots \mathbf{l}_t, \cdots\}$, where $\mathbf{l}_t = \mathbf{l}\mathbf{A}^t$. Note that

$\mathbf{l}_0 = \mathbf{l}$ is the vector of labour directly needed to produce the different products;

$\mathbf{l}_1 = \mathbf{l}\mathbf{A}$ is the vector of quantities of labour directly required in the production of the means of production needed to produce the different products;

$\mathbf{l}_2 = \mathbf{l}\mathbf{A}^2$ is the vector of quantities of labour directly required in the production of the means of production needed to produce the means of production needed to produce the different products;

and so on. Note that vertically integrated labour is equal to the sum of dated labor, i.e. $\mathbf{l}^* = \sum_t \mathbf{l}_t$.

It is easily recognized that prices of production may be also determined by the sum of forward discounted values of dated quantities of labour, i.e.

(34) $\mathbf{p} = w\sum_t \mathbf{l}_t (1+r)^t$,

This *reduction to dated quantities* of labour in particular, or to dated quantities of primary inputs in general, played an important role in the approach of the old Austrians. Eugen von Böhm-Bawerk, Carl Menger, or Knut Wicksell considered production as a time consuming process in which series of inputs of original factors of production, labour and land, result in a final output.

There is, however, a remarkable difference between the Austrian model and the Sraffian approach of representing a price in terms of a reduction to dated labour. In the Sraffian model the series of dated labour is not restricted to be finite.

In order to obtain an Austrian model, we have to impose some technological restrictions, i.e. $\mathbf{A}$ must be such that there is a T for which $\mathbf{A}^T = \mathbf{0}$. This is the case if, by permutation of rows and columns, matrix $\mathbf{A}$ could be brought into a form where the upper diagonal elements are positive, i.e. $\forall a_{ij} > 0$ for $i = j+1$, $j = 1, 2, \cdots n-1$, and all other elements are equal to zero. For such a matrix $\mathbf{A} \in \mathbb{R}^n \Rightarrow \mathbf{A}^n = 0$.[4]

The economic reasoning is the following: The first process, represented by the first column of $\mathbf{A}$ and the first element of $\mathbf{l}$, produces a capital good by unassisted labour. Some amount of this capital good and labour are then used by the second process to produce another capital good. The third process uses some amount of capital goods produced by processes 1 and 2, and so forth. The last process, *n*, uses labour and capital goods produced by some or all other processes and produces pure consumption goods. This concept is also reflected in Carl Menger´s ranking of goods according to their satisfaction of needs and wants. Consumer goods are goods of the first order. Capital goods which enter directly into the production of consumer goods are goods of the second order, a.s.o.

Hicks (1973, p.8) recognized that the flow input-point output model of the old Austrians defined fixed capital away. It is the essential characteristic of 'durable-use goods' that they contribute, not just to one unit of output, at one date, but to a sequence of units of outputs, at a sequence of dates. Retaining the characteristic feature of the Austrian approach, Hicks conceived of production as a process which converts a flow of primary inputs (labour) into a flow of consumer goods. Given a process *k*, determined by $\left(l_0^k, l_1^k, \cdots l_{T_k-1}^k\right) \rightarrow \left(b_1^k, b_2^k, \cdots b_{T_k}^k\right)$, we obtain prices, in terms of 'centre' coefficients for labour.

[4] Hagemann and Kurz (1976) have stressed that there are no basic products in Austrian processes. Note that this is a necessary, but not sufficient condition for an Austrian process.

(36) $$w\frac{\sum_{t=1}^{T_k} l_{t-1}^k (1+r)^{T_k-t}}{\sum_{t=1}^{T_k} b_t^k (1+r)^{T_k-t}}(1+r) = p_k$$

It is straightforward to represent the neo-Austrian process in a Sraffian fashion by a particular pure fixed capital system characterized by the absence of finished capital goods, i.e.

(37) $$\begin{pmatrix} \mathbf{o}^k \\ \mathbf{M}^k \end{pmatrix}, \mathbf{l}^k \rightarrow \begin{pmatrix} \mathbf{b}^k \\ \mathbf{N}^k \end{pmatrix},$$

where $\mathbf{l}^k = \left(l_0^k, l_1^k, \cdots l_{T_k-1}^k\right)$, $\mathbf{b}^k = \left(b_1^k, b_2^k, \cdots b_{T_k}^k\right)$, $\mathbf{o}^k$ is a vector of dimension k containing zero elements and $\mathbf{M}^k, \mathbf{N}^k \in \mathbb{R}^{(T_k-1)\times T_k}$, are matrices as defined in equation (5), Section 2, and represent inputs and outputs of intermediate products

Note that the Austrians or the neo-Austrians do *not* assume that production can be performed by unassisted labour alone. There are capital goods, and there is fixed capital in the neo-Austrian model, but due to total vertical integration it has 'disappeared' from the scene and, therefore, it is possible to represent inputs as consisting of a flow of dated quantities of labour alone. One may represent particular Austrian processes by input and output matrices which reveal the actual participation of circulating and fixed capital[5], but (37) provides the most general representation of a neo-Austrian process.

While the neo-Austrian approach is more general than the Austrian concept, it is still based on the restrictive assumptions that

(i) there is no general joint production, and

(ii) there is no circular production

6. Flows or Funds?

Georgescu-Roegen opens his reflections[6] on production usually with the dictum: 'no analytical boundary, no analytical process'. That analytical boundary has two aspects: First, it contains a temporal component, the duration of the process. Secondly, like a geographical frontier, it separates the process at any point in time from the rest of actuality and, thus, must specify which 'elements', i.e. factors and products, are considered as relevant and must be

[5] See, for instance, Hagemann and Kurz (1976).
[6] See, for instance, Georgescu-Roegen (1971, 1976)

taken into account and which can be neglected. Furthermore, to complete the description of a process, the quantities and the dates of input and output 'elements' are to be specified. So far, there is no difference to the general flow input - flow output concept discussed in Section 2.

Some 'elements' appear *either* as an input *or* as an output of a process. Some inputs, such as non durable produced means of production, maintenance supplies or energy inputs are used up and can never appear as an output of the process in which they are used. Goods and wastes which are produced in a process are not used by the same process. These inputs and all outputs are called 'flow elements'. Note that corn, if it is used as seed, or a hammer used for the production of hammers, do not fall into this category while outputs of corn or hammers do.

Other inputs are not used up; instead they exit the process, in which they participate in an economically, if not physically, identical form and in the same amount. A straightforward example of an element of this category is 'Ricardian Land', which is possessed of 'original and indestructible powers'. The use of land does not involve a change in the quality of the land itself. Land represents a *fund* which delivers some services but, ex hypothesis, cannot be destroyed. Hence it is used, but not in the sense of being consumed. The same applies for labour.

The next problem to be considered is concerned with fixed capital. A machine is a material stock but not in the sense of a stock of raw material which can be used up. Georgescu-Roegen stresses that one may well, and actually should, distinguish between new and used durable capital items and treat the latter as an output of the process in which the new machine is used up. But he hesitates to conceive of a used machine as a commodity which participates in a process.

> Yet in no sense can we say that it is the aim of economic production to produce used tools. Consequently they have no cost of production. Moreover, with the exception of used automobiles and used dwellings, no used capital equipment has a regular market and hence, a price in the same sense in which new equipment has. Used equipment, therefore, is not a commodity proper, and yet no report of a productive process can be complete without reference to it. (Georgescu-Roegen, 1976, p. 83)

Georgescu-Roegen is also aware that computing depreciation of fixed capital by adopting one of the numerous conventions, such as radioactive decay or linear depreciation, involves some arbitrariness and is logically circuitous because it presupposes given prices.

It has been demonstrated in Section 3 that prices of production for used equipment make economic sense and that these prices can be determined without adopting some arbitrary conventions concerning depreciation Hence it seems that Georgescu-Roegen is overly hasty to claim that

> Economic theory has endeavoured to avoid the Gordian knot altogether by building its foundation only upon a process in which *all capital equipment is continuously maintained in its original efficiency*. (Georgescu-Roegen, 1976, p. 83)

and defines

> A fund is both an input and an output; more precisely, it is a factor whose economic efficiency is maintained by the very process in which it participates. ... Ricardian land provides the clearest illustration of the concept of fund, but a machine that is continuously maintained and repaired also fits the definition. Accordingly, a machine coming out of a process is a fund even though it may have no part whatsoever in common with the "same" machine that went into the process. (Georgescu-Roegen, 1976, p. 41).

Two propositions seem to be mixed up here. (i) It is one thing to assume that the efficiency of a machine is constant over the entire period of its life. What else can be meant by *continuously maintained in its original efficiency*? The efficiency of a machine cannot be associated only by the flow of future outputs which can be produced by means of it. A machine which requires more and more inputs to keep outputs constant is of decreasing efficiency. Hence the concept of efficiency involves both the flows of future outputs *and* the flows of future inputs and therefore, a machine is of constant efficiency if and only if the flows of outputs and inputs are constant over its entire life time. (ii) It is, however, another thing to suppose that a machine is maintained such that it can always be considered as being *the 'same' machine that went into the process*. This proposition does not rule out that the efficiency of the machine may vary *during* the process, but supposes that any machine which enters the process as an input must appear, after it has been utilized, as an output. The total amounts of fixed capital items used in a process are therefore equal to the total amounts of fixed capital items 'produced' in that process. Once fixed capital is installed, it can be utilized forever, and the process can be repeated as many times as we want and will obtain the same flow of outputs by using the same flow of inputs, except for the durable means of production which are already in place.

Georgescu-Roegen motivates this peculiar treatment of fixed capital by the classical notion of an economic system being capable of reproduction. This is a viability condition which makes perfect sense for a whole economy, or for a vertical integrated sub-sector of an economy. Georgescu-Roegen is aware of the analytical difficulty of the notion of reproducible process:

> First, in order to maintain the efficiency of a machine intact, we need other machines. Machines and tools to maintain other machines and tools lead to a regress which may stop only if the reproducible process includes practically every production process in the world. (Georgescu-Roegen, 1976, p. 41)

But he insists on the fiction of a partial reproducible process, where fund elements are maintained by outside processes, each one in turn to be analyzed separately.

The proposition neglects a particular type of choice of technique. Keeping the efficiency of installed equipment constant might not be technically feasible, or, if it is, it might not be the most profitable method of utilizing a machine or a building. Depending on the cost of maintenance and repair it might be cost minimizing to abstain from maintenance and let the efficiency of the machine drop until it will be jettisoned.

How does the flow-fund model, and in particular the concept of perennial machines, fit into the framework provided in Section 2? If that framework is general, then the flow-fund model must appear as a special case.

Consider a general flow input-flow output process, where commodities are produced by means of commodities, and, for simplicity, by homogenous labour, i.e.

$$\left(\begin{pmatrix}\mathbf{a}_0\\ l_0\end{pmatrix},\begin{pmatrix}\mathbf{a}_1\\ l_1\end{pmatrix},\cdots\begin{pmatrix}\mathbf{a}_{T_k-1}\\ l_{T_k-1}\end{pmatrix}\right)\rightarrow\left(\mathbf{b}_1,\mathbf{b}_2,\cdots\mathbf{b}_{T_k}\right).$$

Assume that vectors for inputs and outputs are subdivided such that

$$\forall t:\quad \mathbf{a}_t=\mathbf{a}_t^c+\mathbf{a}_t^f,\qquad \mathbf{b}_t=\mathbf{b}_t^q+\mathbf{b}_t^f,$$

where $\mathbf{a}_t^c$ and $\mathbf{a}_t^f$ refer to circulating capital and fixed capital inputs, and $\mathbf{b}_t^q$ and $\mathbf{b}_t^f$ are vectors for products proper and for used capital goods which leave the process at time t. Up to now, there is no additional restrictive assumption. But with perennial machines, being *the 'same' machines that went into the process*, we have the restrictions, that

(i) $\sum_{t=1}^{t}\left(\mathbf{b}_t^f-\mathbf{a}_{t-1}^f\right)\leqq \mathbf{o},\quad t=1,2\cdots T-1$, and

(ii) $\sum_{t=1}^{T}\left(\mathbf{b}_t^f-\mathbf{a}_{t-1}^f\right)=\mathbf{o}$.

Condition (i) states that the sum of old machines ‘produced’ during the interval $[1,t]$ cannot exceed the amount of old machines invested within the interval $[0,t-1]$. Condition (ii) means that at the very end of the process all machines which have been previously invested have left the process unaltered. Note that there are no such restrictions in the general case in which, contrary to the fund approach, durable capital items are differentiated with respect to age.

Georgescu-Roegen decided to represent fund elements not by two flows, one for inputs and one for outputs, but by one single flow of services, representing the amount of flow elements which are actually participating in the process at time t.

Given the assumption of perennial machines formalized by restrictions (i) and (ii), Georgescu-Roegen prefers to represent processes by

$$\left(\begin{pmatrix}\mathbf{a}_0^c\\ \mathbf{h}_0^f\\ l_0\end{pmatrix},\begin{pmatrix}\mathbf{a}_1^c\\ \mathbf{h}_1^f\\ l_1\end{pmatrix},\cdots\begin{pmatrix}\mathbf{a}_{T_k-1}^c\\ \mathbf{h}_{T_k-1}^f\\ l_{T_k-1}\end{pmatrix}\right)\rightarrow\left(\mathbf{b}_1^q,\mathbf{b}_2^q,\cdots\mathbf{b}_{T_k}^q\right),$$

where vectors

$$\mathbf{h}_t^f=\mathbf{a}_0^f+\sum_{t=1}^{t}\left(\mathbf{a}_t^f-\mathbf{b}_t^f\right)\ \text{for}\ t=1,2,\cdots,T-1,\ \text{and}\ \mathbf{h}_0^f=\mathbf{a}_0^f,$$

represent the amounts of perennial capital goods utilized within the interval [t, t+1]. This representation reveals that fixed capital is treated like a natural factor of production. Analogous to the rent on land, the cost of fixed capital is determined by the present value of the rental rates paid for the utilization of machines, or, alternatively by the present value of the difference of the flows of inputs and outputs of these items, i.e.

(38) $$\mathbf{p}_f\sum_{t=1}^{T}\left(\mathbf{b}_t^f-\mathbf{a}_{t-1}^f(1+r)\right)(1+r)^{-t}=\mathbf{r}_f\sum_{t=1}^{T}\mathbf{h}_{t-1}^f(1+r)^{-t},$$

where $\mathbf{p}_f$ and $\mathbf{r}_f$ are prices for fixed capital and corresponding rental rates. Since

(39) $$\sum_{t=1}^{T}\left(\mathbf{b}_t^f-\mathbf{a}_{t-1}^f(1+r)\right)(1+r)^{-t}=r\sum_{t=1}^{T}\mathbf{h}_{t-1}^f(1+r)^{-t},$$

we obtain $\mathbf{r}_f = r\,\mathbf{p}_f$, which are the rental rates for perennial machines.

This peculiar treatment of durable capital is not the only deficiency of the flow-fund aproach. Georgescu-Roegen also subdivides outputs into products proper and wastes and postulates that wastes are flow elements because they do not enter a process. How can that be assured a priori? Consider paper waste, metal scrap or waste glass. There are many examples where wastes, in actual fact, turn into useful products by using recycling processes. Furthermore, a preconceived classifications of products into 'goods' and 'bads' does not make sense. Without taking account of quantities required and the several production processes from which cost-minimizing producers may choose, we cannot subdivide outputs a priori into useful products or wastes, which nobody wants and which are available in excess. If these products can be freely disposed of, then they have neither a value nor do they cause any costs, and, therefore, their prices are zero. If the disposal of outputs available in excess is not free but costly then these products will fetch a negative price.

7. On observable coefficients in input-output models

Comparing Leontief´s input output approach with the theoretical concepts developed by theorists such as Sraffa or von Neumann one must state clearly at the outset that Leontief´s concern was different. As Professor Baumol emphasizes in his paper in the present volume, Wassily Leontief´s contribution should not be viewed as just an incremental additional contribution to the works of earlier scholars such as Quesnay or Marx, but as providing a new and powerful tool applicable to *empirical* analysis. Leontief (1987, p. 860) characterized input-output analysis as 'a practical extension of the classical theory of general interdependence ... to describe and to interpret its operation in terms of directly basic structural relationships'. In accomplishing this task, Leontief gave up some ambitious concepts which can easily be introduced by theorists but which were not empirically applicable because the necessary data are either not accessible or were not available at the time when Leontief started his project.

First of all we will usually find data for inputs and outputs measured in values rather than in quantities. This does not cause any analytical problems. A quantity of a product or a factor can alternatively be measured in tons or in pounds, in liters or in cubic meters. A change of the standard does not cause any analytical trouble as long as it is consistent, i.e. if *all* quantities of a homogenous good are measured in the same way. Hence we may measure quantities in terms of values as long as these values are measured in constant prices.

A more serious problem concerns joint production. It is surprising that input output analysts still define joint production away. Modern input output statistics supply output tables which provide empirical evidence of joint production. In order to adjust the observed data to the input-output model joint production is suppressed by some 'technology assumptions'. Furthermore, if joint products such as pollutants or wastes are dealt with, then the quantities of these by-products are treated as inputs of the polluting process and an abatement process is considered which produces negative quantities of pollutants (Leontief, 1970).

Another problem is that input output models are based on the assumption that there is only one process to produce a certain commodity, i.e. there is no choice of technique. This deficiency combined with the requirement of full utilization of capacities is the reason for negative solutions of the dynamic input-output model. For a discussion of this problem see Lager (1997, pp. 358-359) or Kurz, Dietzenbacher and Lager (1998, pp. *xl-xlii*).

Another problem refers to inhomogeneities. Empirical data of inputs or outputs refer to groups of products or factors. These groups are usually neither homogenous with respect to quality nor homogenous with respect to location. A similar problem concerns the definition of input output sectors. Census data refer to enterprises or to firms which are usually engaged in more than one process, and, consequently, input-output sectors refer to industry groups but not to processes.

The emphasis in this section is on another type of inhomogeneity, i.e. *inhomogeneity with respect to time.* The idea of describing a process by its flows of inputs and outputs as outlined above is a powerful tool for theoretical reasoning but one will hardly find data for a rigorous application of that concept. Observable data on inputs or outputs consist either in the sum of quantities produced or used during a period of time (year or month) or in stocks of inputs or outputs at the beginning of that period of accounting, which has, in general, nothing to do with the uniform period of production of point input point-point output processes or with the duration of a flow input-flow output process.

Consider a flow input-flow output process of duration T , which produces a flow of products being homogenous with respect to quality (and location). Assume that this process has been activated at time $t-1,\ t-2,\ \cdots t-T$ at intensities $x_{t-1},\ x_{t-2},\ \cdots x_{t-T}$. Set aside the problem of choice of technique, and, therefore the possibility of truncation. In this case total output of products of these processes available at time t is

(40) $$q_t = \sum_{t=1}^{T} b_t x_{t-t} .$$

Inputs of products used by these processes at time t, denoted by vector $\mathbf{u}_t$, consist of all inputs of finished products of these processes initiated at time t, $t-1$, $\cdots t-(T-1)$, i.e.

(41) $$\mathbf{u}_t = \sum_{t=0}^{T-1} \mathbf{a}_t x_{t-t} .$$

We cannot observe outputs or inputs at a point in time but we can observe quantities of outputs or inputs produced or used during a ‘year’. Assume that the ‘year’ is of arbitrary length d. Hence we observe the *rate of the flow per ‘year’* for outputs

(42) $$\tilde{q}_{[t_0,d]} = \sum_{t=t_0}^{t_0+d} q_t = \sum_{t=t_0}^{t_0+d} \sum_{t=1}^{T} b_t x_{t-t} ,$$

and for inputs

(43) $$\tilde{\mathbf{u}}_{[t_0,d]} = \sum_{t=t_0}^{t_0+d} \mathbf{u}_t = \sum_{t=t_0}^{t_0+d} \sum_{t=0}^{T-1} \mathbf{a}_t x_{t-t} .$$

Using these observed rates of inputs and outputs per ‘year’ to calculate flow input-flow output coefficients we obtain a vector of *observable input output coefficients.*

(44) $$\tilde{\mathbf{a}}_{[t_0,d]} = \tilde{\mathbf{u}}_{[t_0,d]} \left(\tilde{q}_{[t_0,d]} \right)^{-1} = \sum_{t=t_0}^{t_0+d} \sum_{t=0}^{T-1} \mathbf{a}_t x_{t-t} \left(\sum_{t=t_0}^{t_0+d} \sum_{t=1}^{T} b_t x_{t-t} \right)^{-1}$$

Note that these observable coefficients are not only determined by the technology but are also influenced by the intensities at which the respective processes have been initiated. If and only if the observed economy is stationary, i.e. $x = x_t = x_{t-t}$, one may obtain ‘technical’ flow-flow coefficients.

If we can assume that the economy (or the sector) grows with a constant rate $\bar{g}$ per time unit, and, therefore, $x_{t-t} = x_t \left(1+\bar{g}\right)^{-t}$, we obtain

(45) $$\tilde{\mathbf{a}}_{[t_0,d]} = \sum_{t=0}^{T} \mathbf{a}_t \left(1+\bar{g}\right)^{-t} \left(\sum_{t=1}^{T} b_t \left(1+\bar{g}\right)^{-t} \right)^{-1} ,$$

which depend on technical parameters $\mathbf{a}_t$, b_t and on the rate of growth $\overline{g}$. Note that these coefficients can be used for a quantity model which is based on the assumption of proportional growth at rate $\overline{g}$ but cannot be used in the case that the rate of growth $g \neq \overline{g}$ or for a price model where, in general, the rate of profit $r \neq \overline{g}$.

For some circulating capital inputs such as raw materials or energy and for some labour inputs it might be reasonable to assume *'strict proportionality'* between inputs and outputs, i.e.

$$a_t = a^* b_{t+s} \quad \text{for} \quad \forall t \leq (T - s), \quad \text{and} \quad a_t = 0 \quad \text{for} \quad \forall t > (T - s),$$

where scalars a_t denote quantities of some inputs and s is the length of the production lag. In this case we obtain

$$\text{(46)} \qquad \tilde{a}_{[t_0,d]} = a^* \frac{\sum_{t=0}^{T-s} b_{t+s} \left(1+\overline{g}\right)^{-t}}{\sum_{t=s}^{T} b_t \left(1+\overline{g}\right)^{-t}} = a^* \left(1+\overline{g}\right)^s .$$

Hence, if there is 'strict proportionality' between inputs and outputs, if the economy, or the sector grows with a constant rate, and if the production lag is small such that the growth factor can be neglected we may approximate technical coefficients by observed flow-flow coefficients.

Strict proportionality might be a reasonable assumption for some inputs but cannot be assumed for spare parts, tools or, in general, for fixed capital inputs. For the latter inputs Leontief proposed to use stock-flow coefficients.

The time profile of fixed capital inputs will, in general, depend on the life time of the durable items and on the construction lag (see Lager, 1997). Assume, for simplicity, that a_0 quantities of a machine are invested at the beginning of the production process and that the lifetime of the machine is equal to the duration of the flow input-flow output process. Assume furthermore that the quality of the machine does not alter during its entire life and, therefore, the quantities of outputs produced by the machine remain constant. Hence we may add up the quantities of all machines invested during the period $\left[(t_0 + 1 - T), t_0\right]$ and obtain stocks of machines available at time t_0 , i.e.

$$\text{(47)} \qquad s_{t_0} = a_0 \sum_{t=0}^{T-1} x_{t_0 - t} \; .$$

Given the stocks of machines, we may calculate stock-flow coefficients (for a constant rate of growth)

(48) $$c_{[t_0,d]}=\frac{a_0\sum_{t=0}^{T-1}x_{t_0-t}}{b\sum_{t=t_0}^{t_0+d}\sum_{t=1}^{T}x_{t-t}}=\frac{a_0}{b}\;\frac{1-\left(1+\overline{g}\right)^{-T}}{\left(1+\overline{g}\right)^{d}-1}\,.$$

Note that the stock-flow coefficients also depend crucially on the dynamics of the economy. But while the flow-flow coefficients are independent of the length of the 'year' the stock-flow coefficients decrease with the length of the accounting period. The latter fact, which has been recognized by Leontief (1970), raises some doubts about the validity of that concept.

Finally it shall be outlined that the flow input-flow output approach discussed in Section 1 might be also empirically applicable.

Given

(i) the rates of flows of inputs and outputs during the entire duration of the process[7], i.e. $\tilde{\mathbf{u}}_{[t_0,T]}$ and $\tilde{\mathbf{q}}_{[t_0,T]}$,

(ii) an estimate for the average rate of growth, g^*, per time unit, which is assumed to be steady, and

(iii) a preconceived opinion about the temporal pattern of the flows of inputs and outputs in that sector, denoted by $\left(\mathbf{a}_0^*,\mathbf{a}_1^*,\cdots\mathbf{a}_{T-1}^*\right)$ and $\left(\mathbf{b}_1^*,\mathbf{b}_2^*,\cdots\mathbf{b}_T^*\right)$,

and set aside technical changes, one may obtain estimates $\mathbf{b}_t^\circ=\widehat{\mathbf{m}}\,\mathbf{b}_t^*$ and $\mathbf{a}_t^\circ=\hat{\mathbf{n}}\,\mathbf{a}_t^*$ for outputs and inputs of the flow input-flow output process such that

(49) $$\tilde{\mathbf{q}}_{[t_0,T]}=\widehat{\mathbf{m}}\sum_{t=1}^{T}\mathbf{b}_t^*\left(1+g^*\right)^{-t}$$

and

(50) $$\tilde{\mathbf{a}}_{[t_0,T]}=\hat{\mathbf{n}}\sum_{t=0}^{T-1}\mathbf{a}_t^*\left(1+g^*\right)^{-t}\,,$$

[7] For most circulating capital goods and for labour we may reasonably base our preconceived opinion on the 'strict proportionality' assumption mentioned above. In this case it will suffice to use rates of flows per calendar year. The life of most durable items will exceed that period. Hence we must base our estimation procedure on a period which equals at least the life of the fixed capital item in question.

where $\widehat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ are diagonal matrices made from vectors **m** and **n** which can be determined by using available data.

In actual fact, we cannot avoid to take technical changes and changing rates of growth into account. Therefore, it seems to be clear that the approach sketched above, is far away from actual applicability and must be seen as a first suggestion for the empirical implementation of a production model based on flow input-flow output coefficients.

8. Concluding remarks

Starting from a rather comprehensive view of production characterised by general flow input-flow output processes, it has been demonstrated that the Sraffa - von Neumann method of total vertical disintegration and the representation of technology by matrices for inputs and outputs does not require any additional restrictive assumptions. The other extreme, i.e. the totally vertically integrated view of production, and the representation of processes by flows of dated primary inputs and flows of dated outputs is, in principle, an equivalent method as long as it is not presupposed that these flows are finite and that the vertically integrated processes can be viewed as being independent from each other. Georgescu-Roegen´s assumption that fixed capital items are continuously maintained in its original efficiency implies that the duration of fixed capital is infinite and neglects the possibility of truncation. With regard to waste it has been argued that one cannot presuppose a subdivision of products into 'goods' and 'bads'. Finally it has been demonstrated, that the statistical basis for input-output models involves some intricacies and deficiencies. Observable flow-flow coefficients and stock-flow coefficients cannot be viewed as 'technical' coefficients, but depend, in general, on the intensities at which processes are activated. Finally a solution to that problem is sketched.

References

Böhm-Bawerk, E. von (1889). *Kapital und Kapitalzins. Zweite Abteilung: Positive Theorie des Kapitals*. Innsbruck: Wagner. 4th edn 1921, Jena: Fischer.

Burmeister, E. (1974). "Synthesysing the Neo-Austrian and Alternative Approaches to Capital Theory: A Survey," *Journal of Economic Literature*, **12**, pp. 413-56.

Georgescu-Roegen, N. (1971).). *The Entropy Law and the Economic Progress*. Cambridge, Mass: Harvard University Press

Georgescu-Roegen, N. (1976).). *Energy and Economic Myths*. New York/Toronto/Oxford: Pergamon Press.

Hagemann, H. and Kurz, H. D. (1976). "The Return of the Same Truncation Period and Reswitching of Techniques in Neo-Austrian and More General Models," *Kyklos*, **29**, pp. 678-708.

Hicks, J. R. (1970). "A Neo Austrian Growth Theory," *Economic Journal*, **80**, pp. 257-81.

Hicks, J. R. (1973). *Capital and Time: A Neo-Austrian Theory*. Oxford: Oxford University Press

Kurz, H. D. and Salvadori, N. (1995). *Theory of Production - A Long Period Analysis*. Cambridge, UK: Cambridge University Press.

Kurz, H. D. and Salvadori, N. (1999). "Production and Economic Theory: Reflections on Georgescu-Roegens Contribution," to be published.

Kurz, H. D., Dietzenbacher, E. and Lager, C. (1998). *Input-Output Analysis*. *Vol I*. Cheltenham, UK: Edward Elgar.

Lager, C. (1997). "Treatment of Fixed Capital in the Sraffian Framework and in the Theory of Input-Output Analysis," *Economic Systems Researchl*, **9**, pp. 357-73.

Leontief, W. (1970). "The Dynamic Inverse," in: Carter, A. P. and Brody, A. (eds.), *Contributions to Input Output Analysis*. Amsterdam: North-Holan Publishing, pp. 17-46.

Leontief, W. (1987). "Input-Output Analysis," in: Eatwell, J., Milgate, M. and Newman, P. (eds.), *The New Palgrave*. London and Basingstoke: MacMillan, pp. 860-864.

Neumann, J. von (1937). "Über ein ökonomisches Gleichungssystem und eine

Verallgemeinerung des Brouwerschen Fixpunktsatzes," *Ergebnisse eines mathematischen Kolloquiums*, **8**, pp. 73-83.

Neumann, J. von (1945). "A Model of General Economic Equilibrium," *Review of Economic Studies*, **13**, pp. 1-9.

Schefold, B. (1971). *Mr. Sraffa on Joint Production*. Ph.D. thesis, University of Basel, mimeo.

Schefold, B. (1989). *Mr. Sraffa on Joint Production and Other Essays*. London: Unwin Hyman.

Sraffa, P. (1960). *Production of Commodities by Means of Commodities. Prelude to a Critique of Economic Theory*. Cambridge, UK: Cambridge University Press.

Tani, P. (1988). "Flows, Funds, and Sectorial Interdependence in the Theory of Production," *political economy - Studies in the Surplus Approach*, **4**, pp. 3-21.

Wicksell, K. (1934). *Lectures on Political Economy.* Vol I. London: George Routledge and Sons.